HOMEWORK
From Dieter
1-1, 1-2, 1-3, 1-5, 1-7, 1-8, 3-1

Module #1

States of Stress and Strain

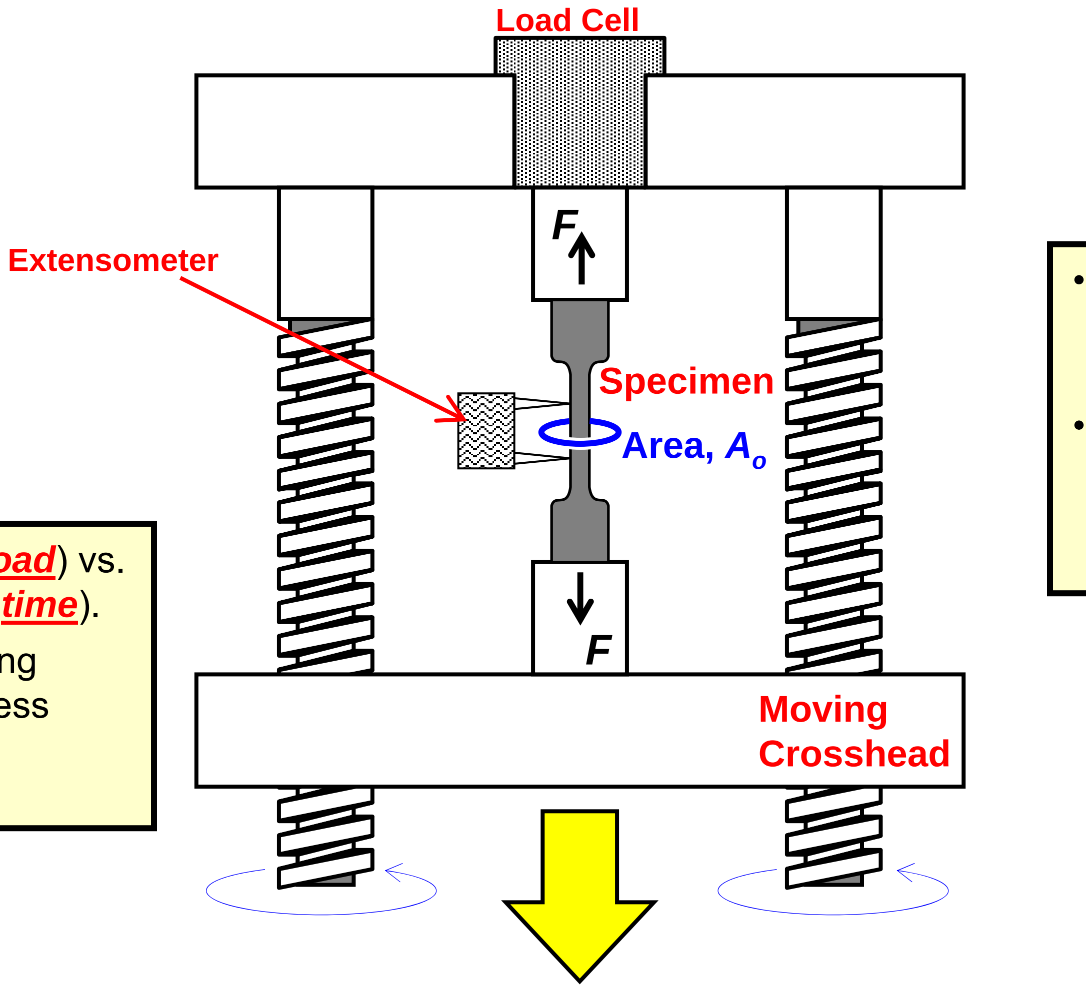
READING LIST

DIETER: Ch. 1, pp. 7-17; Ch. 2, pp. 18-20 and 31-36; Ch. 3 pp. 70-76; Ch. 8 pp. 275-289

Ch. 2 in Meyers & Chawla, 1st ed.
Ch. 2 in Roesler et al.
Ch. 2 in Courtney
Ch. 5 in Nye
Ch. 2 in McClintock and Argon

Tensile Test

The most common way to assess the mechanical behavior of a material (strength and ductility)

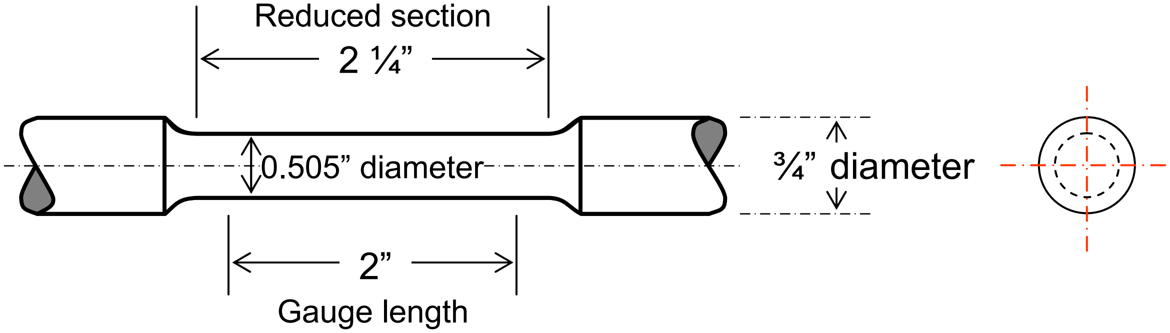


- Used mostly for metals and polymers.
- Not used for ceramics except at very high temperatures.

- Collect ***force*** (or ***load***) vs. ***displacement*** (or ***time***).
- We use the resulting information to assess “strength” and “deformability”

Tensile Test Specimen

A standard tensile specimen



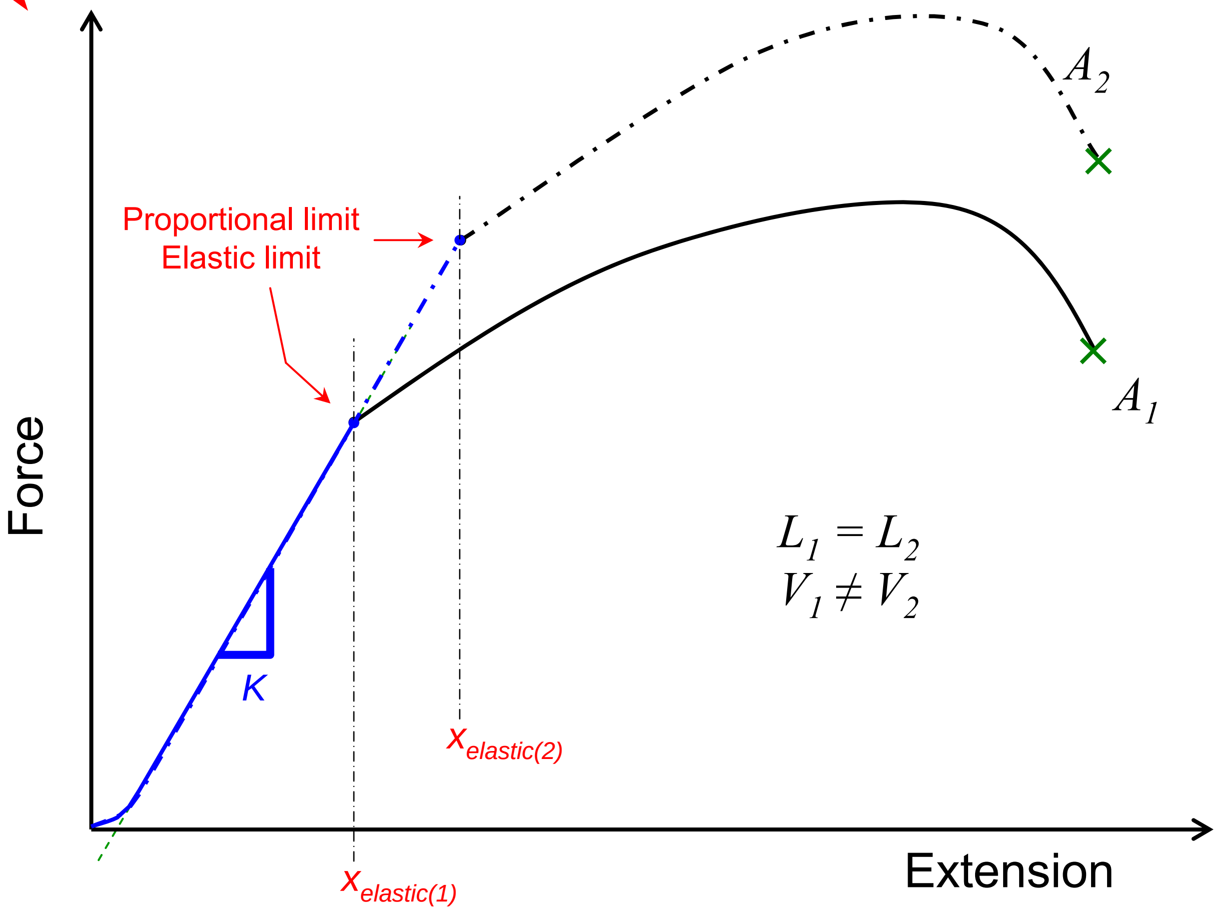
Standard Geometries

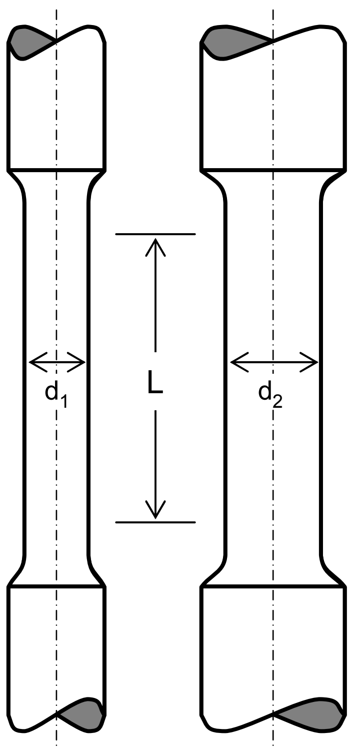
“Buttonhead” – circular cross-section

“Dogbone” – flat cross-section

ASTM E8 or D638

Force - Extension Curve





When $x \leq x_{elastic}$, $F = K\,x$

When $x > x_{elastic}$, $F \neq K\,x$

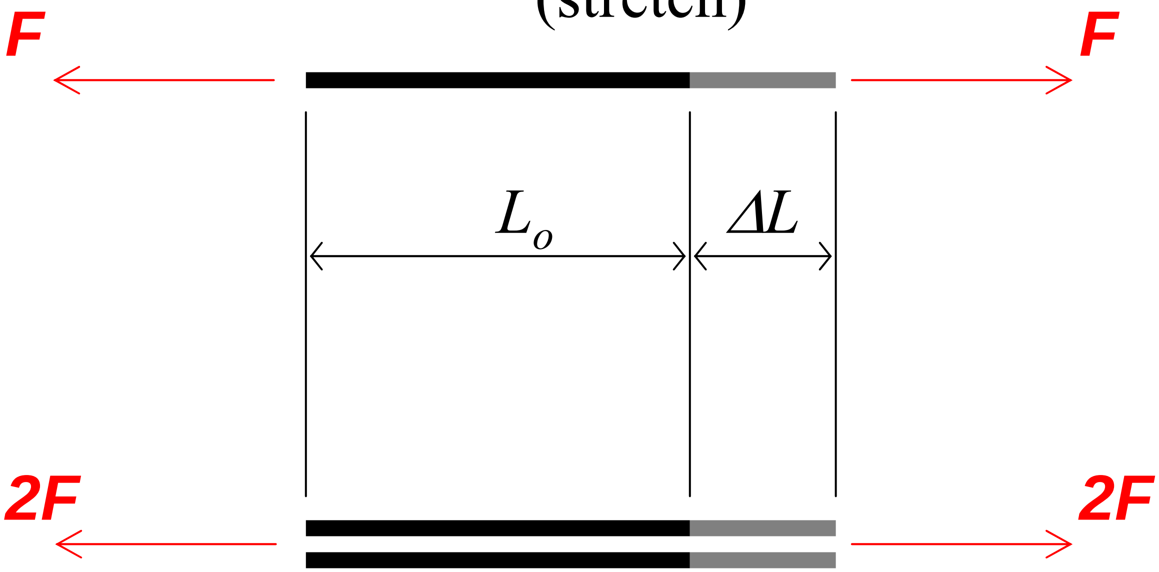
As deforming volume changes, the force to deform the specimen changes. **WHY?**

FORCE

ONE ELASTIC ROD

the force required to deform it to failure is *F*.

(stretch)



TWO IDENTICAL ELASTIC RODS, CONNECTED TOGETHER

the force required to deform them to failure is 2*F*

★

The amount of force needed to deform a solid depends on the volume or surface area of the body.

★ Engineering Stress

$$\sigma_E = s = \frac{F}{A_o} = \frac{\text{applied load}}{\text{original cross-sectional area}}$$

Units for stress and conversion factors

- Load per unit area (“force distribution”)
  - PSI: 1 lb/in^2 = 6.895 x 10^{-3} MPa
    = 7.032 x 10^{-4} kg/mm^2
    = 6.8 x 10^4 dynes/cm^2
  - MPa: 1 MPa = 1 MN/m^2 = 1 N/mm^2 = 1 x 10^6 N/m^2
- An *engineering stress*, as defined above, represents the *average stress* in the object.

As defined above, this represents a normal tensile stress.

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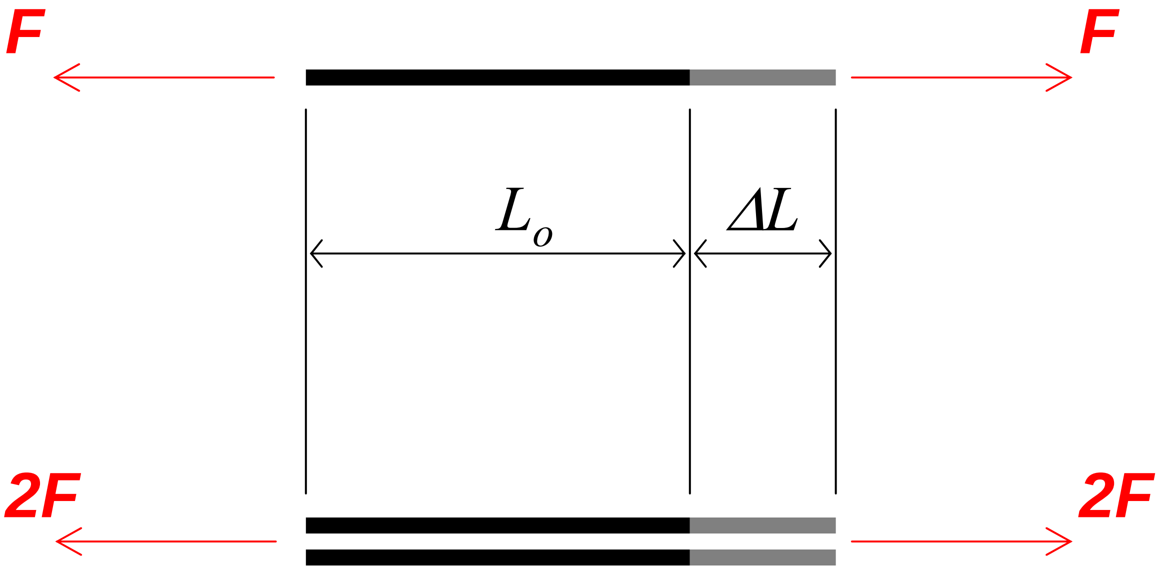
TENSILE STRESS

CONSIDER A SINGLE ELASTIC ROD

the stress required to deform it to failure is

$\sigma = F/A$



IF TWO IDENTICAL RODS ARE JOINED AND DEFORMED

the stress required to deform them to failure is

$\sigma = 2F/2A = F/A$

The amount of STRESS needed to deform a solid does not depend upon the volume or surface area of the body.

Engineering Strain

$$\varepsilon_E = e = \frac{L_i - L_o}{L_o} = \frac{\Delta L}{L_o} = \frac{\text{change in length}}{\text{initial length}}$$

- Length per unit length
  - We express strain either as a fraction or as a percentage. Be careful when doing homework or solving real engineering problems.
  - Ex:
    - *e* = 0.02 is the same as 2% strain
    - *e* = 0.10 is the same as 10% strain
    - *e* = 1.00 is the same as 100% strain

- *Engineering strain*, as defined above, represents the *average linear strain* in the object.

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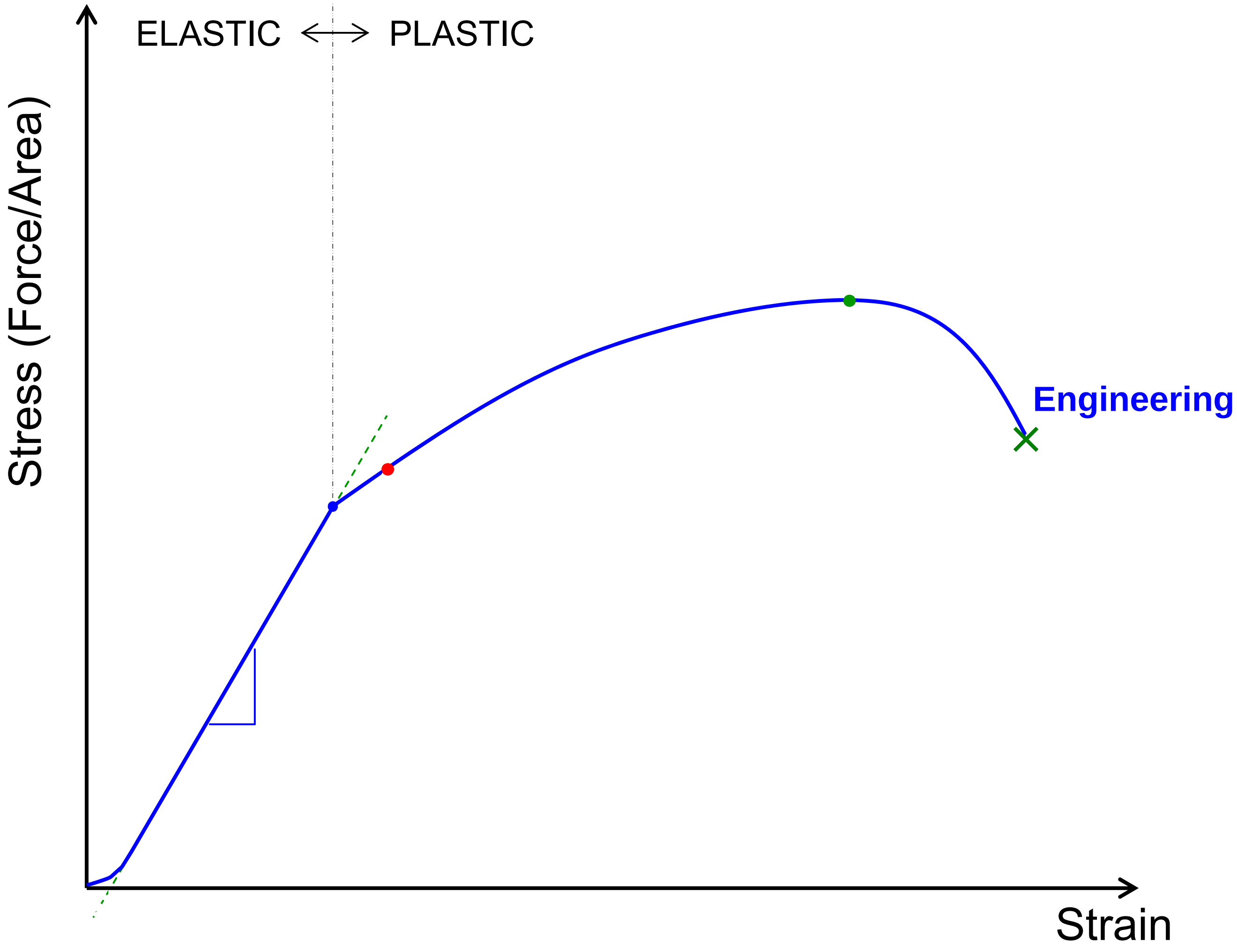
STRAIN IS UNITLESS!

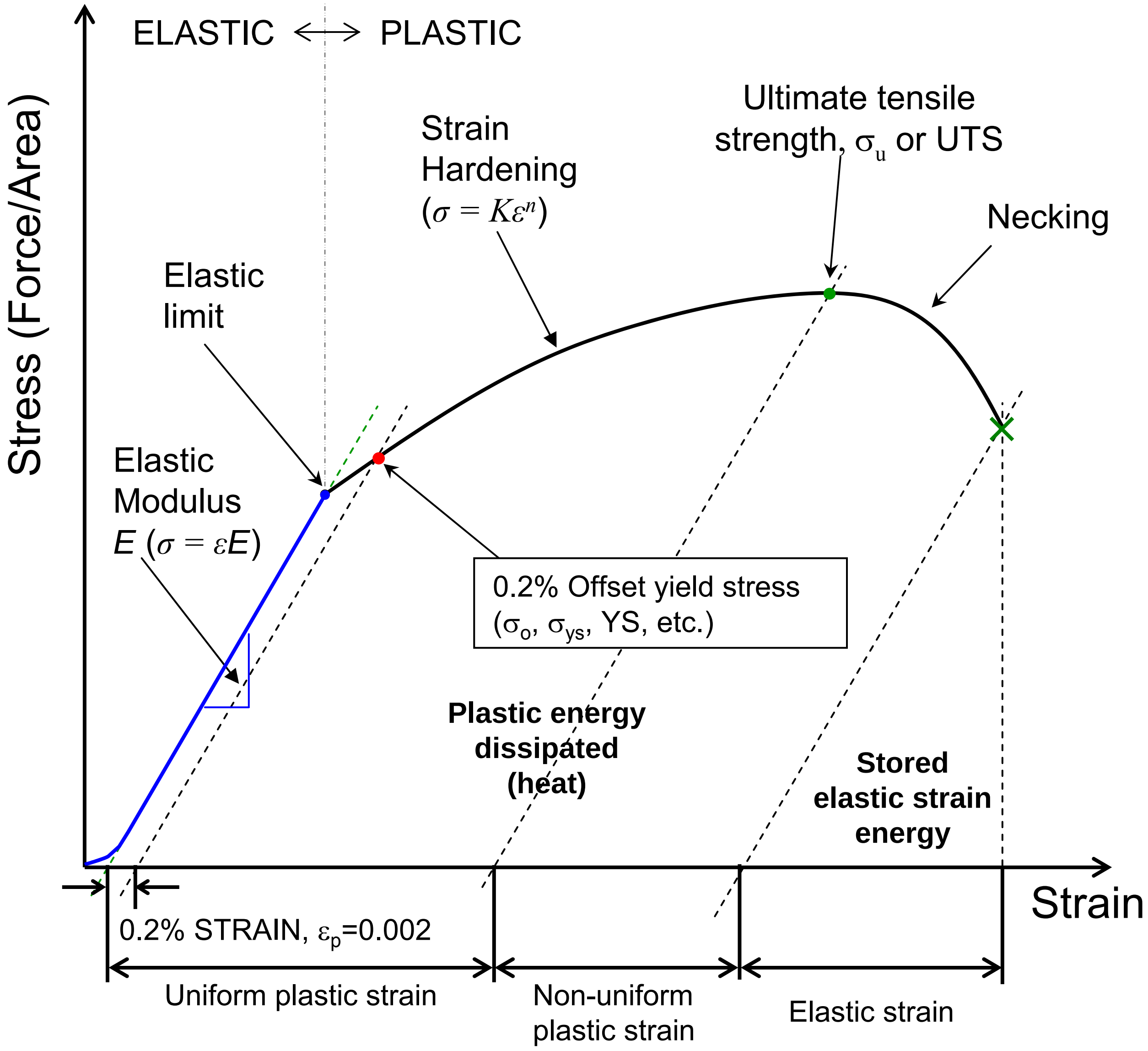
It is a measure of the amount of distortion (i.e., “deformation”) caused by the application of a force.

★ Engineering Stress-Strain Curve in Tension



A*

Engineering Stress-Strain Curve in Tension



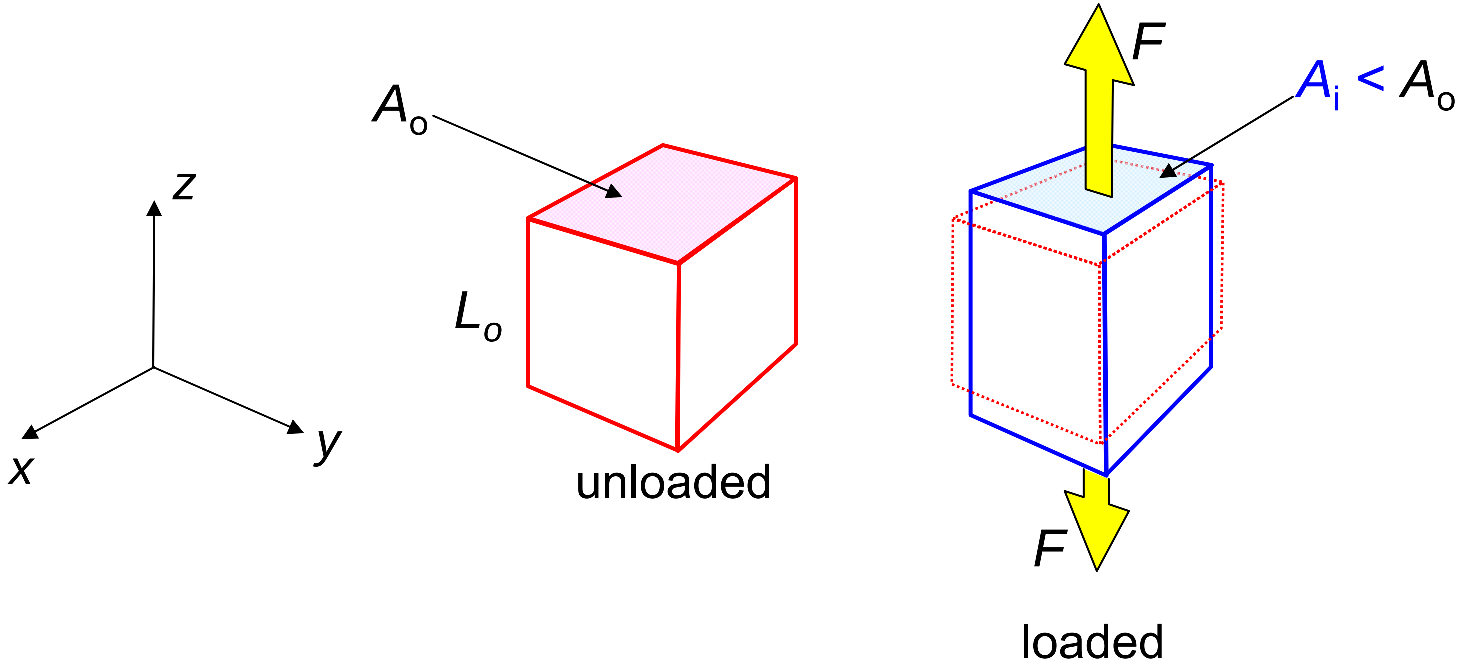
- Elastic deformation up to elastic limit.
- Plastic deformation after elastic limit.
- <u>Uniform plastic deformation</u> between elastic limit and the UTS.
- Nonuniform plastic deformation <u>after</u> UTS.
- In tension this non-uniform deformation is called necking.

B*

★ True Stress – True Strain

- *Volume is* generally *conserved* during deformation.
  Thus "shape changes with deformation."

► *Re-define stress and strain* to account for shape change.



- *True stress*, σ_T = load / instantaneous area = F/A_i
- *True strain*, ε_T = ???

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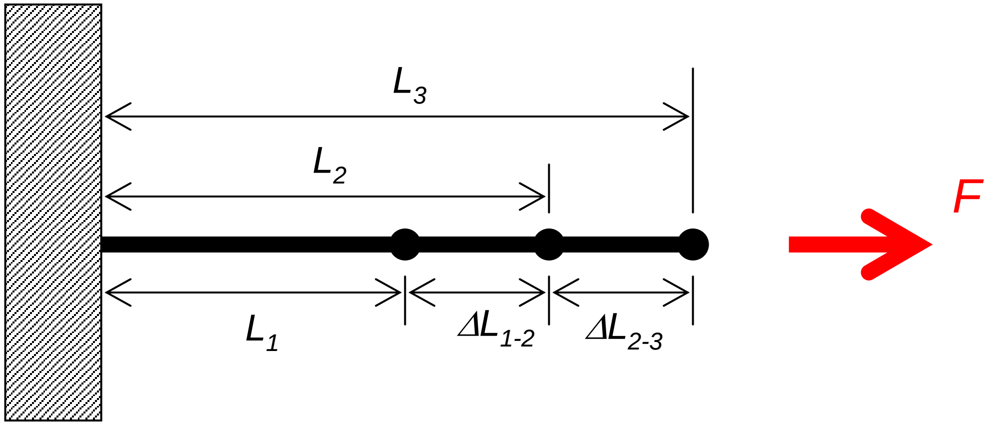
Why true strain?



- *Engineering strain* is the *average linear strain* in the solid.
- *Doesn't work* if we consider *shape change during deformation*.

$$e = \frac{\Delta L}{L_o} = \frac{L_i - L_o}{L_o}$$

$$\therefore$$

$$\left.\begin{aligned} e_{1-2} &= \frac{\Delta L_{1-2}}{L_1} = \frac{L_2 - L_1}{L_1} \\ e_{2-3} &= \frac{\Delta L_{2-3}}{L_2} = \frac{L_3 - L_2}{L_2} \end{aligned}\right\} e_{1-3} = \frac{\Delta L_{1-3}}{L_1} = \frac{L_3 - L_1}{L_1} \neq e_{1-2} + e_{2-3}$$

★

- An incremental displacement results in an infinitesimal strain:

$$d\varepsilon = \frac{\text{change in length}}{\text{instantaneous length}} = \frac{dL}{L}$$

- *Re-define strain* by *integrating* the *infinitesimal displacements* from the *initial to* the *final length*.

$$\boxed{\varepsilon_T \;=\; \text{true strain} = \int_{L_o}^{L_f} \frac{dL}{L} = \ln\left(\frac{L_f}{L_o}\right)}$$

$$\therefore$$

$$\left.\begin{aligned} \varepsilon_{1-2} &= \ln\left(\frac{L_2}{L_1}\right) \\ \varepsilon_{2-3} &= \ln\left(\frac{L_3}{L_2}\right) \end{aligned}\right\} \varepsilon_{1-3} = \ln\left(\frac{L_3}{L_1}\right) = \varepsilon_{1-2} + \varepsilon_{2-3} = \ln\left(\frac{L_2}{L_1}\right) + \ln\left(\frac{L_3}{L_2}\right) = \ln\left(\frac{L_3}{L_1}\right)$$

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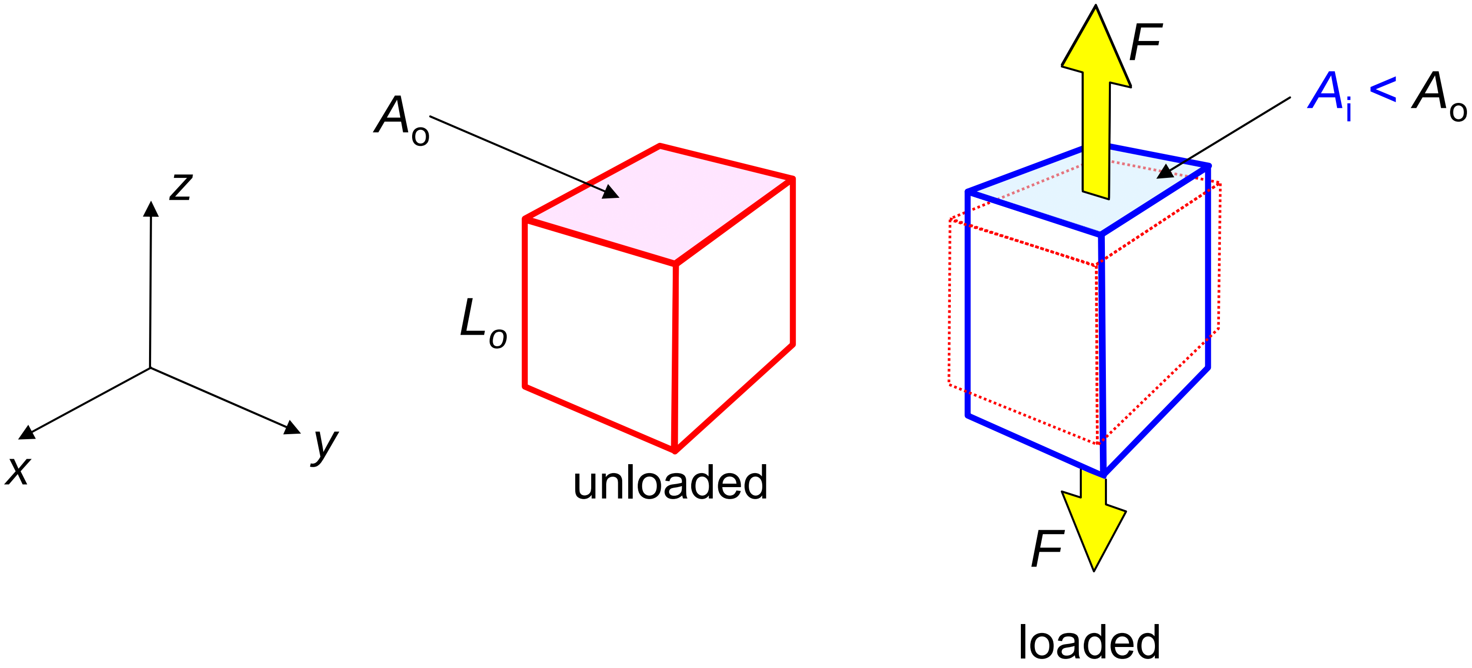
True Stress – True Strain

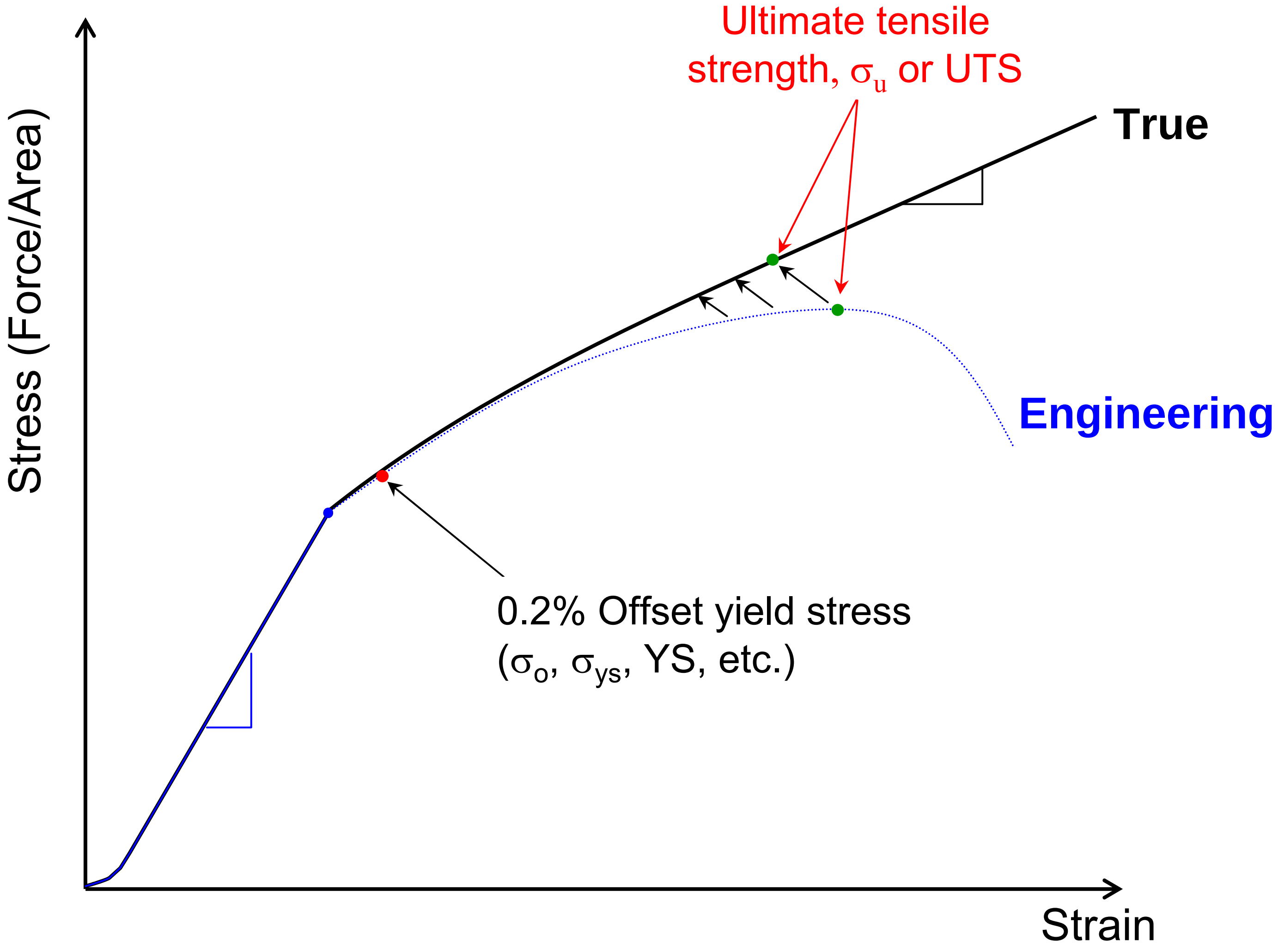


True stress, $\sigma_T = \text{load/instantaneous area} = F/A_i = \sigma_E(\varepsilon_E + 1)$

True strain, $\varepsilon_T = \dfrac{\Delta L}{L_i} = \displaystyle\int_{L_o}^{L_f} \frac{dL}{L} = \ln\left(\frac{L_f}{L_o}\right) = \ln\left(\varepsilon_E + 1\right)$

Use true strain when
structure = Fcn(ε)

True Stress-Strain Curve in Tension



- True stress-strain curve shifts up and to the left of engineering stress-strain curve.

Definitions and relationships between true and engineering stress and strain

Parameter	Fundamental Definition	Before necking	After necking ★
Engineering stress, σ_E	$\sigma_E = \frac{F}{A_o}$	$\sigma_E = \frac{F}{A_o}$	$\sigma_E = \frac{F}{A_o}$
True stress, σ_T	$\sigma_T = \frac{F}{A_i}$	$\sigma_T = \frac{F}{A_i}$ $= \sigma_E \left(1 + \varepsilon_E\right)$	$\sigma_T = \frac{F}{A_{neck}}$
Engineering strain, ε_E	$\varepsilon_E = \frac{\Delta L}{L_o}$	$\varepsilon_E = \frac{\Delta L}{L_o}$	$\varepsilon_E = \frac{\Delta L}{L_o}$
True strain, ε_T	$\varepsilon_T = \ln \frac{A_o}{A_{\min}}$	$\varepsilon_E = \ln \frac{L_i}{L_o}$ $= \ln \frac{A_o}{A_i}$ $= \ln\left(1 + \varepsilon_E\right)$	$\varepsilon_T = \ln \frac{A_o}{A_{neck}}$

★

Remarks about σ and ε

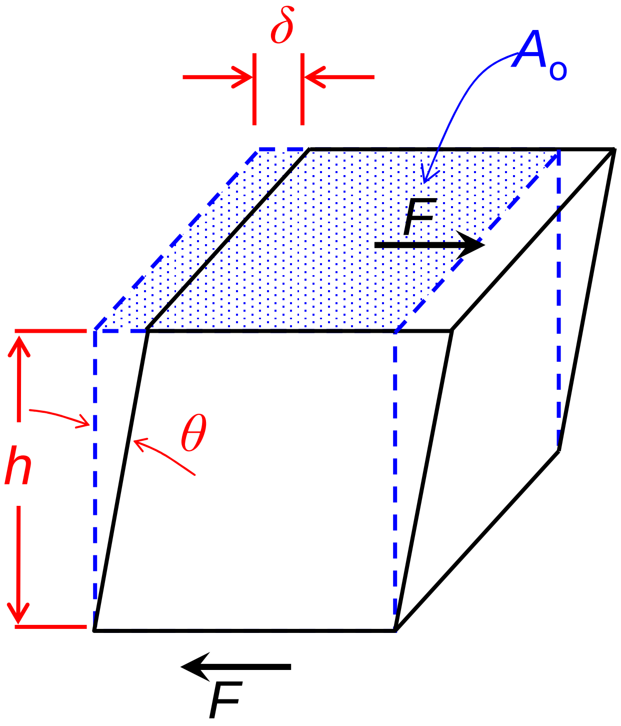
1. For small amounts of deformation (i.e., elastic), engineering and true stresses and strains are the same.

2. They deviate at high strains.

3. Result: true stress and strain are used in metal working operations.

4. Most handbook values (used for design) are engineering stress and strain.

Shear Stress and Strain

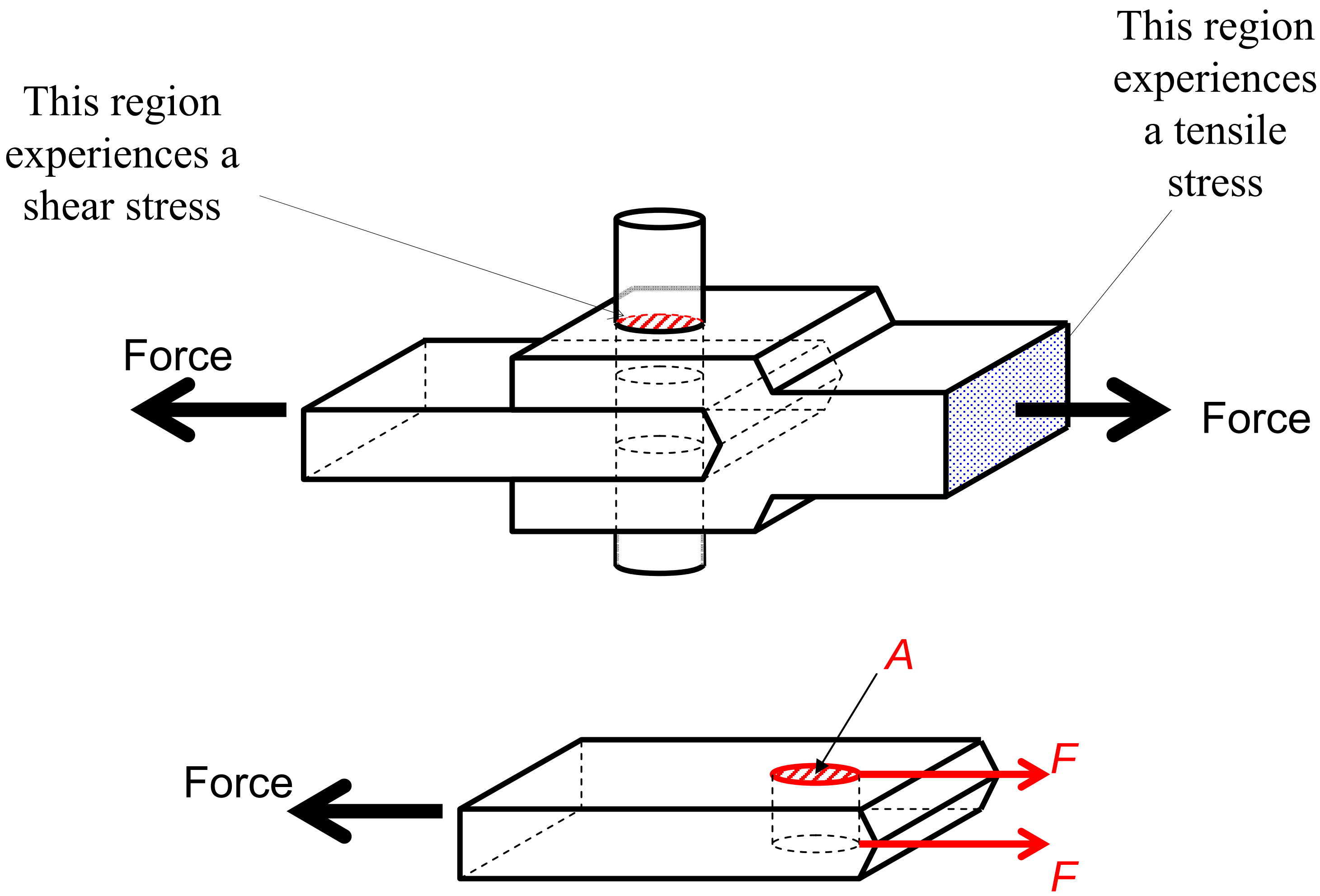


$$\tau = \frac{F}{A_o}$$

$$\gamma = \frac{\delta}{h} = \tan\theta$$

In a shear stress, a force is distributed within a planar area

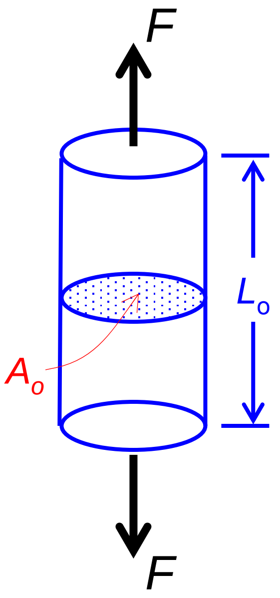
Shear stresses distort objects
Shear strains are a measure of shear distortion



This region experiences a shear stress

This region experiences a tensile stress

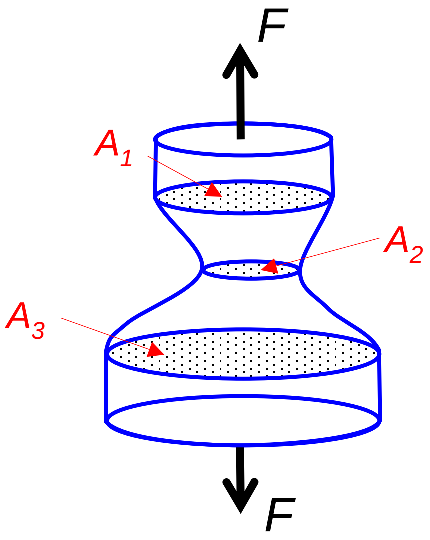
In general the stress required depends on the area of interest and/or the direction of loading



A

Round rod in tension.
Stress is uniform throughout
the structure.

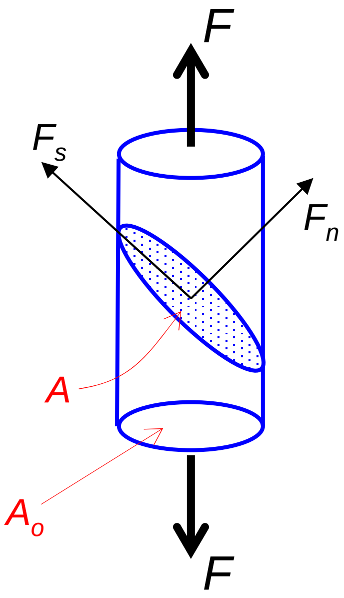
Tensile stress: $\sigma = F/A_o$



B

Non-uniform structure in tension.
Stress varies with position.

Tensile stress at 1: $\sigma = F/A_1$
Tensile stress at 2: $\sigma = F/A_2$
Tensile stress at 3: $\sigma = F/A_3$



C

Area of interest not
perpendicular to load.
Stress can be re-defined
relative to a preferred
coordinate system.

Tensile stress: $\sigma = F/A_o$
Shear stress: $\tau = F_s/A$
Normal stress: $\sigma = F_n/A$

★ Thermal Stresses and Strains

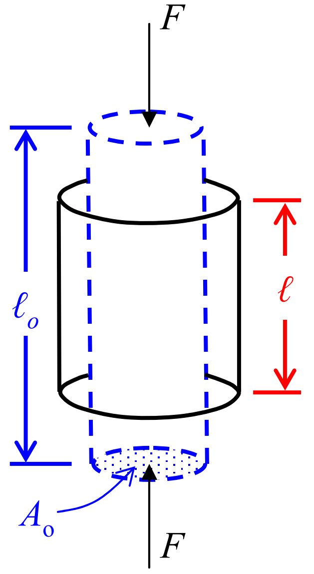
- Most engineering materials expand when heated and contract when cooled.
- The strain caused by a change of one degree (1°) in temperature is known as the coefficient of thermal expansion (α).
- The strain caused by temperature change ΔT is:

$$\varepsilon_t = \alpha \Delta T$$

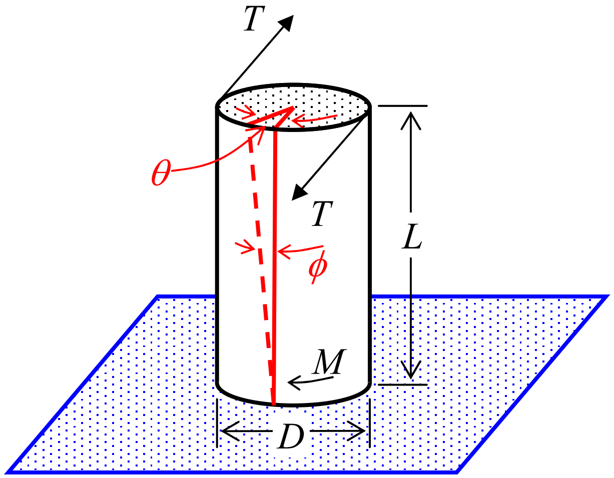
$$\sigma_t = E\varepsilon_t$$

Modes of Deformation Besides Tension

COMPRESSION



TORSION



$$\tau_{\max} = \frac{Tr}{J};$$

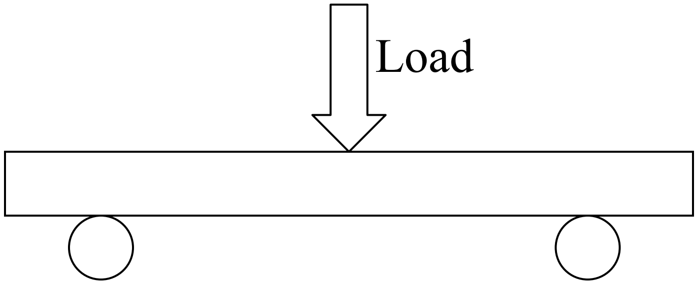
$T =$ torque;

$r =$ radius of cylinder

$J =$ polar moment of inertia $= \dfrac{\pi r^4}{2}$

Modes of Deformation Besides Tension

Flexure/Bend Testing



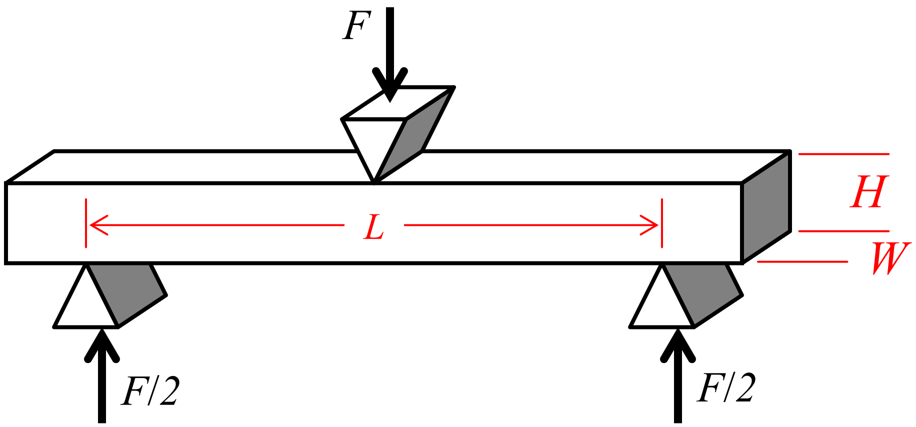
- Commonly used with brittle materials that behave in a linear elastic manner (Ex, ceramics and glasses).
- Governed by two equations:

$$\frac{M}{I} = \frac{E}{R} \text{ and } \frac{M}{I} = \frac{\sigma}{y}$$

- M = applied bending moment,
- I = second moment of inertia of the beam about the neutral plane,
- E = Young's modulus,
- R = radius of curvature,
- σ = tensile or compressive stress
- y = planar distance from the neutral plane. $y = \frac{FL^3}{48EI}$

Flexure/Bend Testing

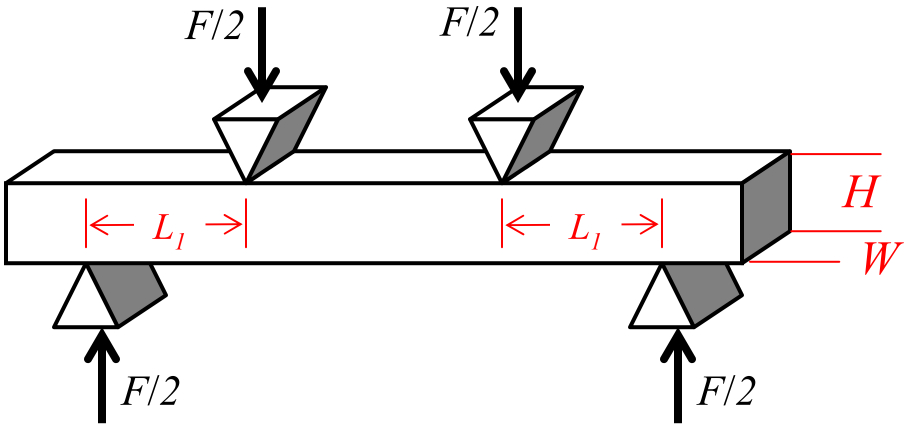
3-Point



$$\sigma_{\max} = \frac{FLH}{8I}$$

$$I_{\text{rectangular}} = \frac{WH^3}{12}\,;\, I_{\text{round}} = \frac{\pi D^4}{64}$$

4-Point



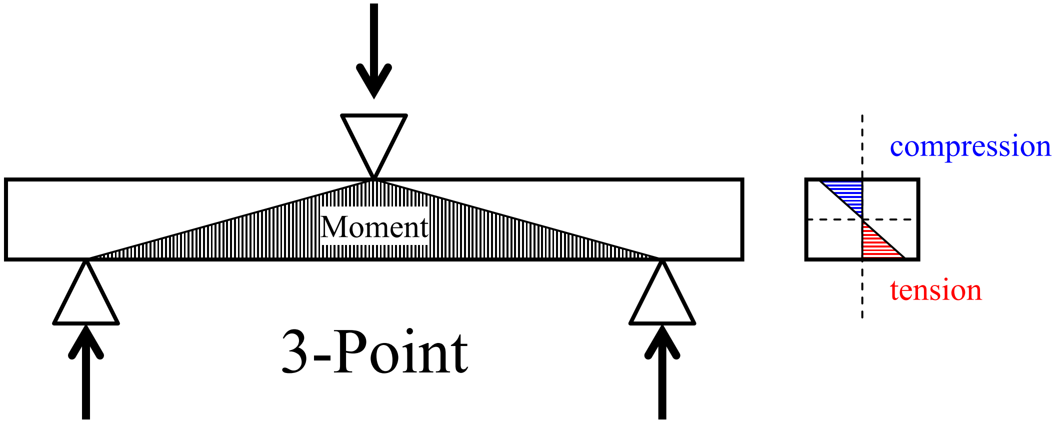
$$\sigma_{\max} = \frac{FLH}{4I}$$

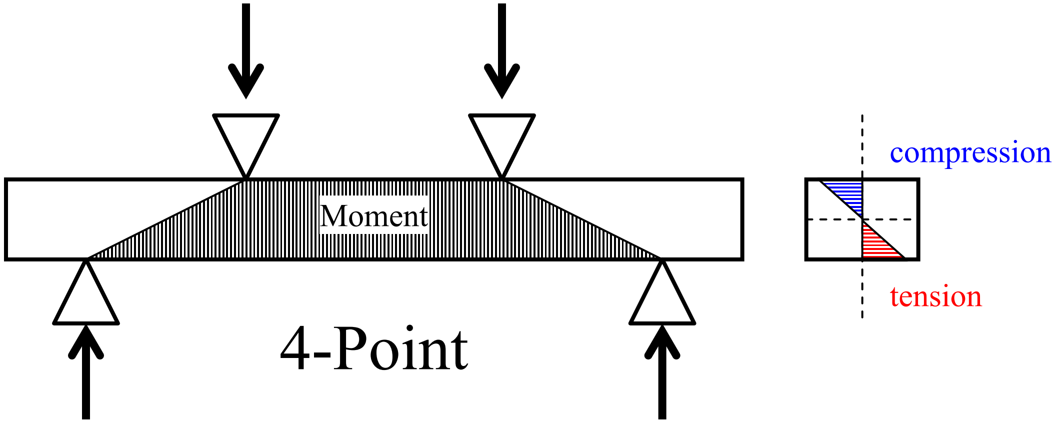
$$I_{\text{rectangular}} = \frac{WH^3}{12}\,;\, I_{\text{round}} = \frac{\pi D^4}{64}$$

σ_{max} is the Modulus of Rupture

$I_{\text{rectangular}}$ and I_{round} represent the moments of inertia for uniform rectangular or round cross-sections.

Moment Diagrams and Stress Distributions in Modulus of Rupture Tests





Be careful when comparing fracture results from tension, 3-point bending and 4-point bending tests as the volume of material at maximum stress is higher in 4-point bending.

4-Point vs. 3-Point Bend Tests

- In four-point bend tests, the material over the inner span is subjected to a constant stress; whereas in three-point bend tests, stresses are localized on the top surface.

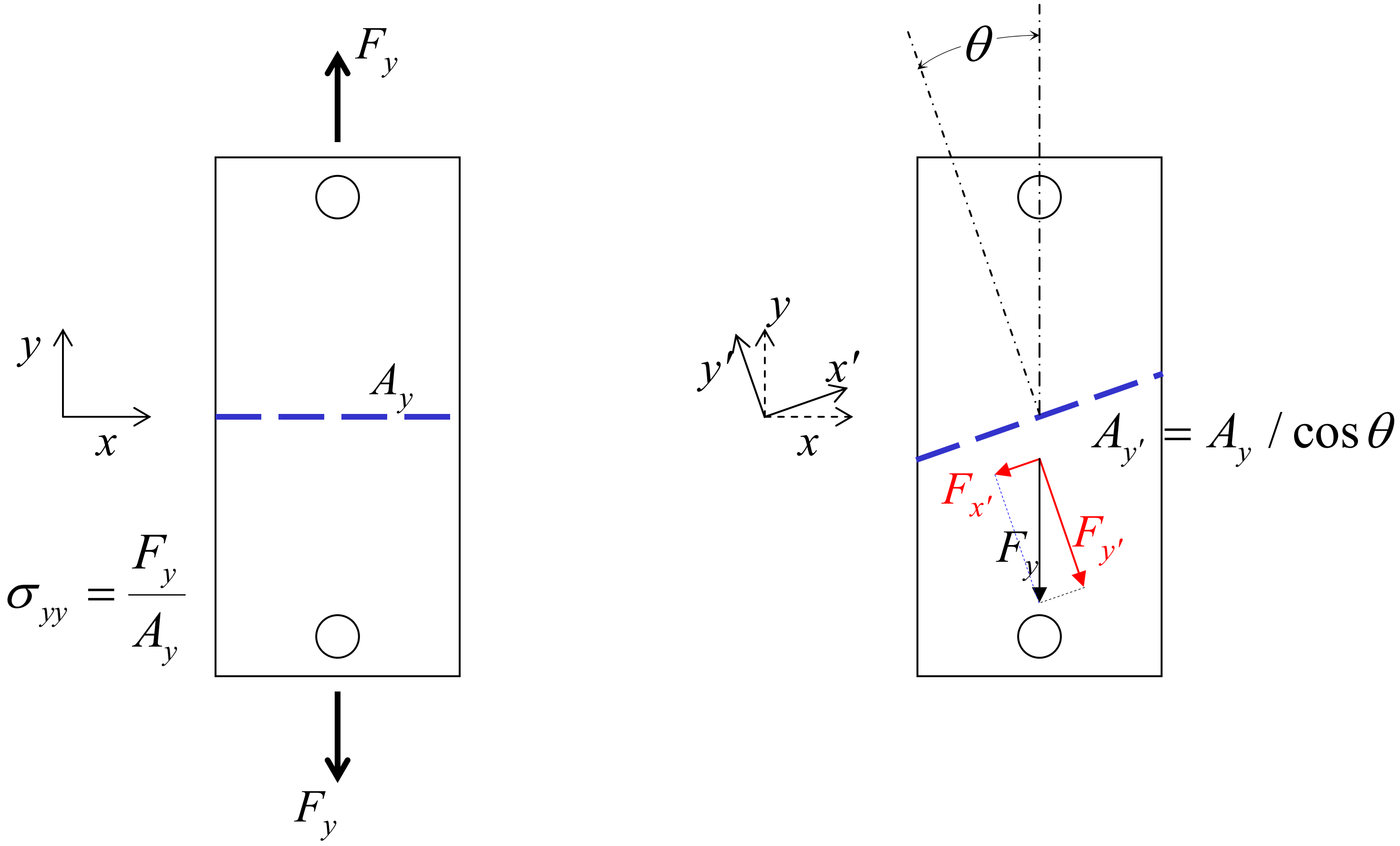
- Thus a larger volume of material is tested in 4-point bend tests than in 3-point bend test. As a result, materials flaws are more likely to reside in the region of maximum stress in a 4-point bend test.

FOR THE TIME BEING, WE WILL CONCENTRATE ON TENSILE AND COMPRESSIVE MODES OF LOADING

5 minute break

State of Stress

Most easily described by normal and shear stress components.



Define relative to feature of interest.

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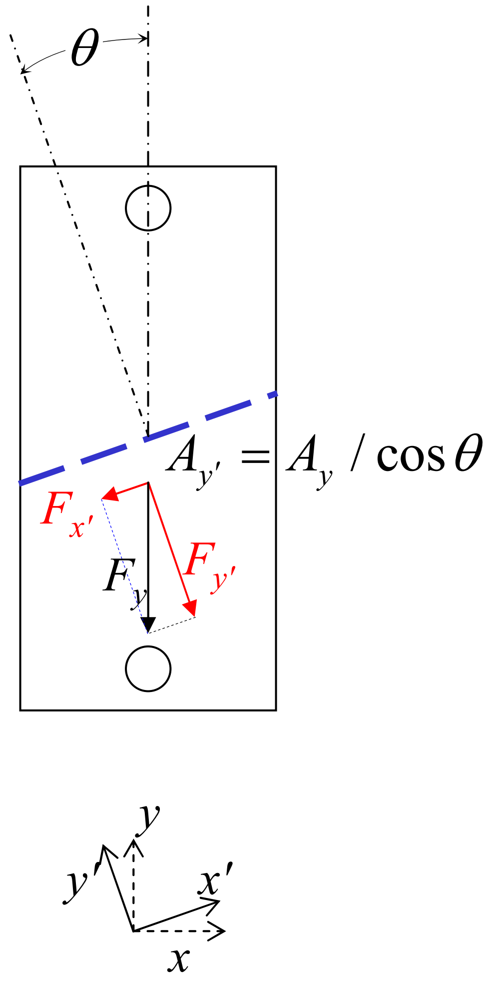
- Normal Stress

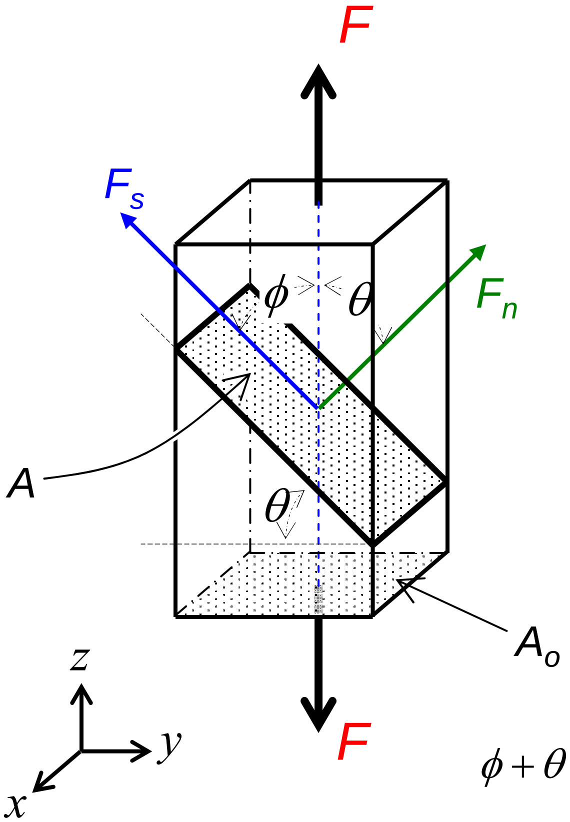
$$\sigma_{y'y'} = \frac{F_{y'}}{A_{y'}} = \frac{F_y \cos\theta}{A_y / \cos\theta} = \sigma_{yy} \cos^2\theta = \frac{\sigma_{yy}}{2}(1 - \cos 2\theta)$$

- Shear Stress

$$\tau_{y'x'} = \frac{F_{x'}}{A_{y'}} = \frac{F_y \sin\theta}{A_y / \cos\theta} = \sigma_{yy} \sin\theta \cos\theta = \frac{\sigma_{yy}}{2} \sin 2\theta$$



Total stress (F) resolved onto an oblique plane



$\phi + \theta = 90°$

$$\sigma_n = \frac{F_n}{A} = \frac{F\sin\phi}{\left(A_o/\cos\theta\right)} = \frac{F}{A_o}\cos^2\theta$$

$$\tau_s = \frac{F_s}{A} = \frac{F\cos\phi}{\left(A_o/\cos\theta\right)} = \frac{F}{A_o}\cos\theta\sin\theta$$

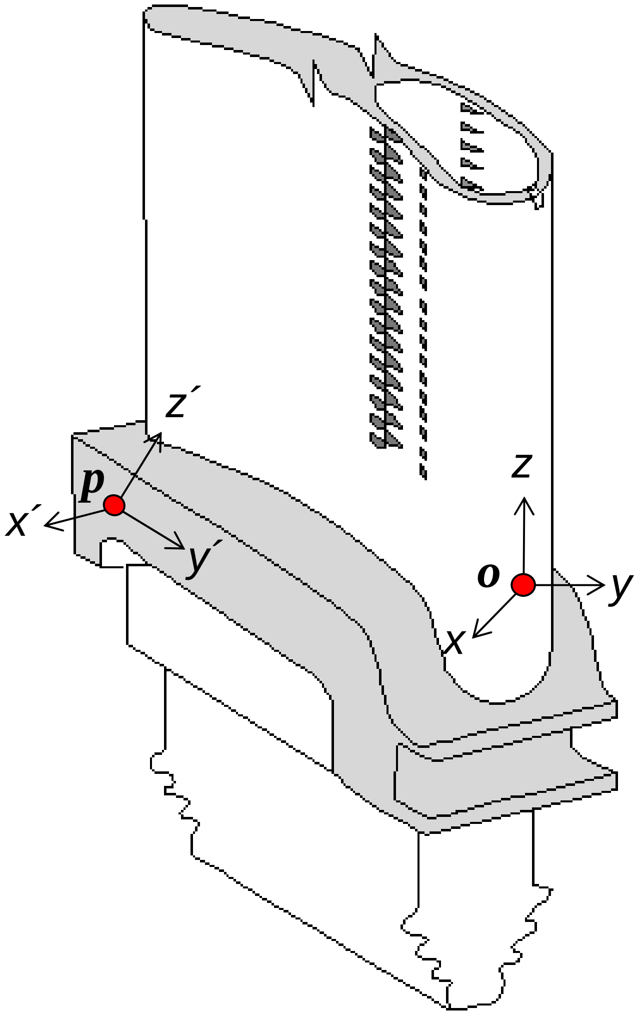
States of Stress & Strain

In most service conditions and forming operations, loading is not uniaxial.

Most engineering structures and materials experience *multiaxial loading*.

This means that they will be subject to varying combinations of
normal AND *shear*
stresses.

How do we define stress and strain under these conditions?



Multiaxial loading is implied from changing geometries of real components.

Stress states in vary from point to point throughout the entire object.

We can easily describe the state of stress using extensions of what we have learned already.

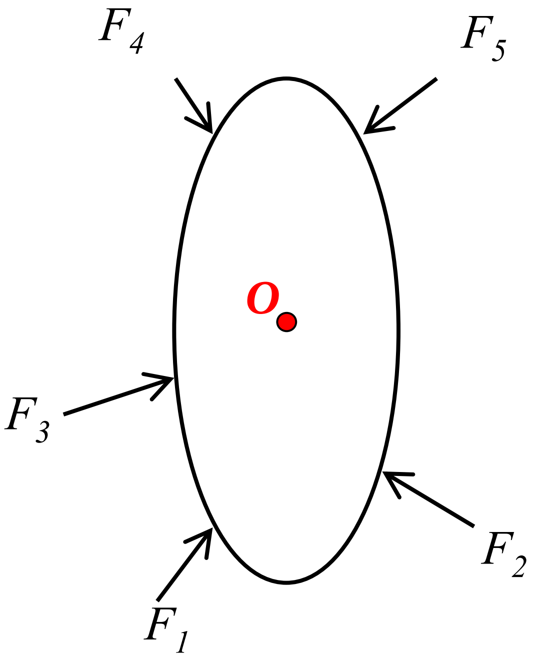
States of Stress

- It is easiest to define the distribution of forces (and thus the distribution of stresses) relative to a planar area within the object.

- NORMAL forces or stresses that are ⊥ to the plane.

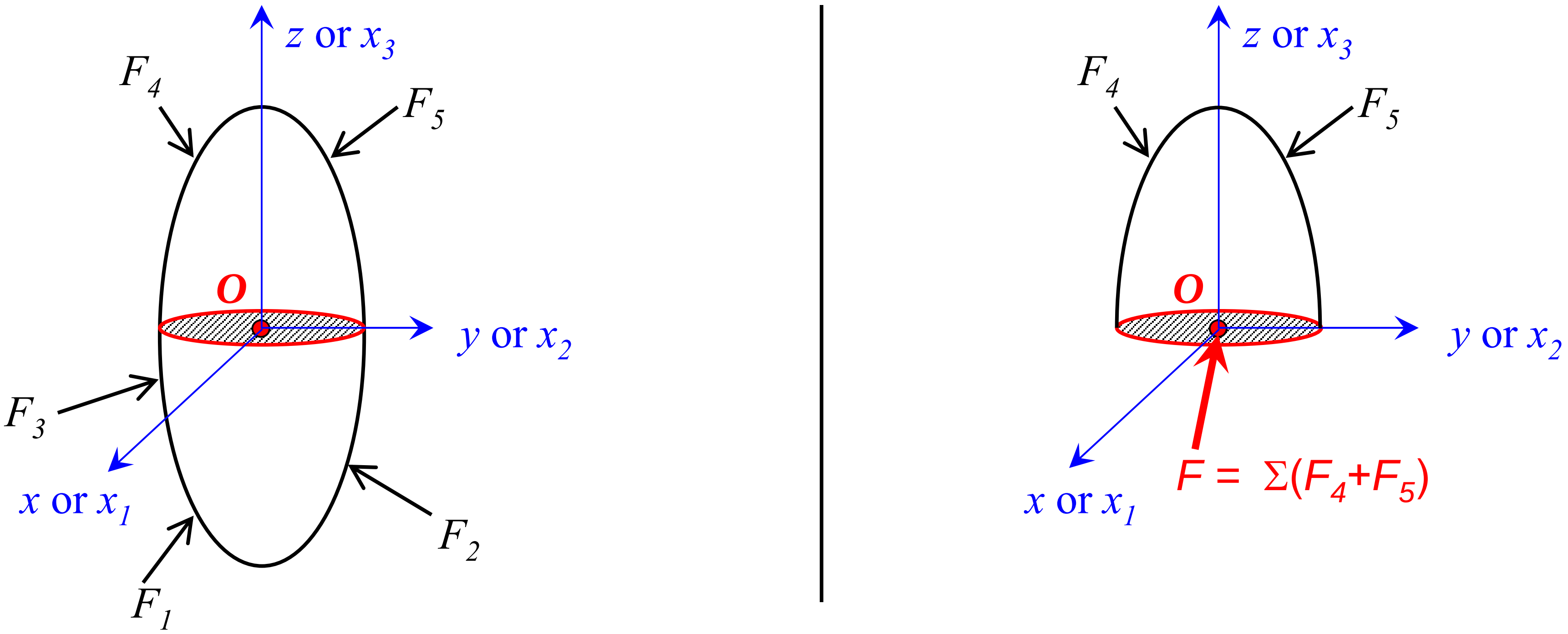
- SHEAR forces or stresses that are // to the plane.

- The relationships that we will develop will allow us to define the state of stress relative to any coordinate system. Pay attention! *Think!*

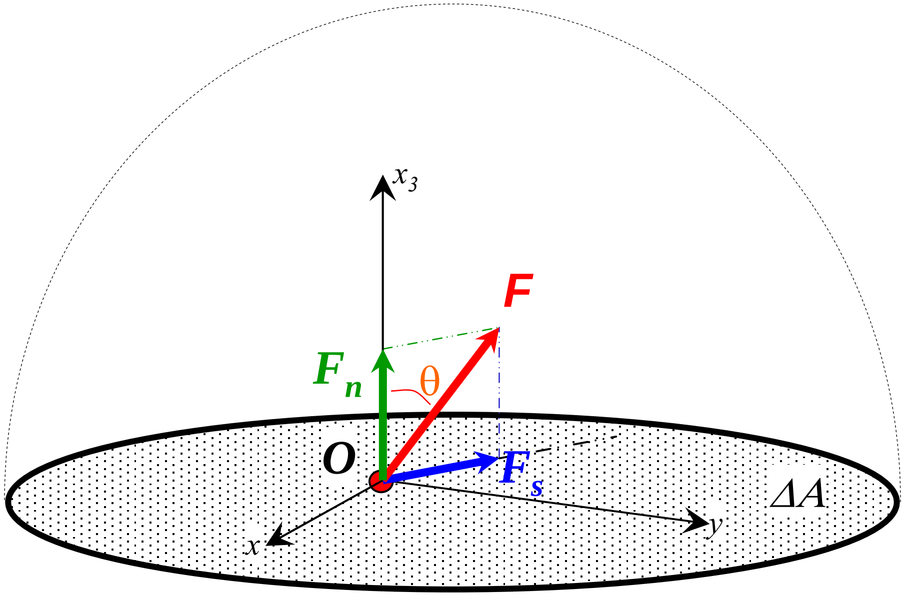


How do we define the state of stress at point ***O***?

- Consider an arbitrary solid object that has a series of external forces applied to it.
- Assume that the object remains *in static equilibrium*.
- Forces applied on the exterior of the solid are balanced by internal tractions (i.e., internal forces) that keep the object in equilibrium (i.e., keep it from moving or changing shape).
- Thus the solid is under a state of stress.



- Define a planar area that passes through the point (it can be placed anywhere) and establish an orthogonal (3-D) coordinate system passing through the point where one axis is the normal to the plane and the other two axes lie within the plane.
- Sum forces such that the resultant force *F* on the point keeps the body from moving. [ΔF=0 at equilibrium]
- This is easiest to visualize if you section the body at the plane, ignore one half, and place a resultant force *F* on the plane such that it opposes the surface forces acting on body.



Resolution of force *F* into normal and shear force components relative to a planar area

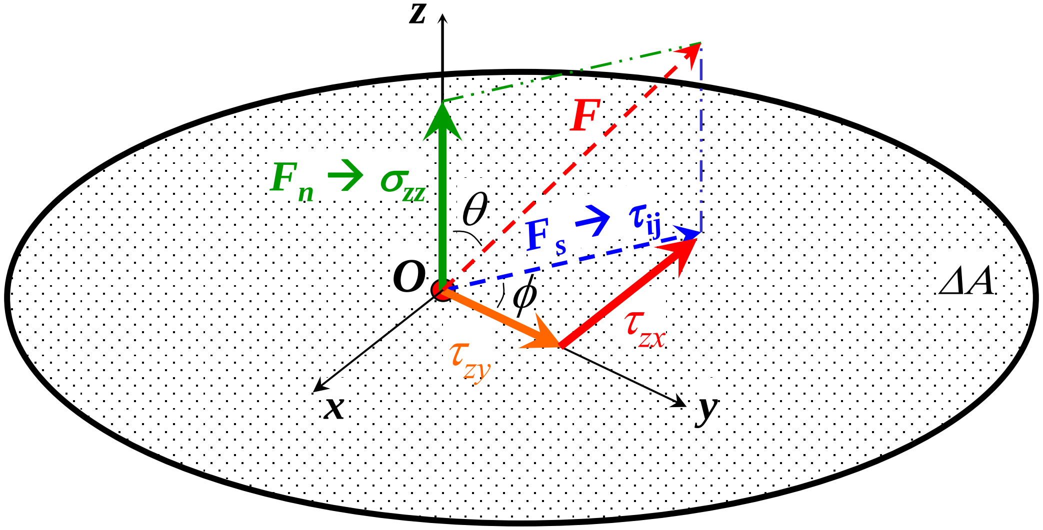
- Focusing on the plane and a small area (ΔA) surrounding point O, the resultant force *F* can be resolved into normal, F_n, and shear, F_s, components relative to the plane and our orthogonal coordinate system.

- The components of force can, in turn, be converted into stresses.

$$\text{Stress at Point } O = \lim_{\Delta A \to 0} \frac{F}{\Delta A}$$

- Each stress component can be related to our coordinate system.

Conversion of force F into stress components acting on a plane



NORMAL: $\sigma_{ii} = \sigma_{zz} = \frac{F_n}{A} = \frac{F\cos\theta}{A}$

SHEAR: $\tau_{ij} = \frac{F_s}{A} = \frac{F\sin\theta}{A}$

Parallel to y-dir. $\rightarrow$ $\tau_{zy} = \tau_{32} = \frac{F_y}{A} = \frac{F_s\cos\phi}{A} = \frac{(F\sin\theta)\cos\phi}{A}$

Parallel to x-dir. $\rightarrow$ $\tau_{zx} = \tau_{31} = \frac{F_x}{A} = \frac{F_s\sin\phi}{A} = \frac{(F\sin\theta)\sin\phi}{A}$

RELATIVE TO ANY PLANE,

The state of stress at any point can be defined by one NORMAL STRESS and two SHEAR STRESSES.

The normal and shear components can be conveniently related to an orthogonal coordinate system.

★

Notation for Stresses

$$\sigma_{ij}\ ;\ \tau_{ij}$$

In this course we shall refer to our subscripts* as follows:

First Subscript:

Corresponds to the *plane* that the *stress acts upon*.

Second Subscript:

Corresponds to the *direction* that the *stress* component *is pointed in*.

*Many texts reverse the meaning for each subscript.

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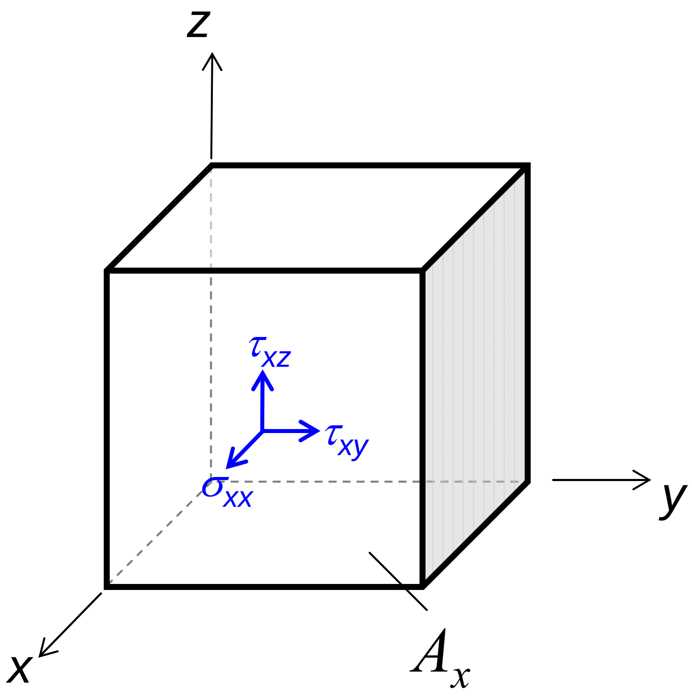
Definition of stresses relative to a planar area and a 3-D coordinate system



Normal stress: $\sigma_{xx} = \frac{F_x}{A_x}$

Shear stress: $\tau_{xy} = \frac{F_y}{A_x}$

$\tau_{xz} = \frac{F_z}{A_x}$

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Sign Convention for Stresses

Normal Stresses:

Positive – tension

Ex., $\sigma_{zz} = +100\ MPa$

Negative – compression

Ex., $\sigma_{zz} = -100\ MPa$

Shear Stresses:

Positive – acts on (+) face & points in (+) direction

Ex., σ_{zy} or τ_{zy}

Negative – acts on (-) face & points in a (+) direction

Ex., $\sigma_{z,-y}$ or $\tau_{z,-y}$

NOTE: With shear stresses, sometimes a σ is used rather than a τ. Watch your subscripts.

Stress at a Point

- Normal stress at a point

$$\sigma_{ii}(i = x, y, z) \text{ normal to area } \delta A \text{ at a point is}$$

$$\sigma_{ii} = \lim_{\delta A \to 0} \frac{\delta F_{y'}}{\delta A}$$

- Shear stress at a point

$$\tau_{ij} = \sigma_{ij}(i \neq j = x, y, z) \text{ parallel to area } \delta A \text{ at a point is}$$

$$\tau_{ij} = \sigma_{ij} = \lim_{\delta A \to 0} \frac{\delta F_{x'}}{\delta A}$$

★

The Stress Tensor

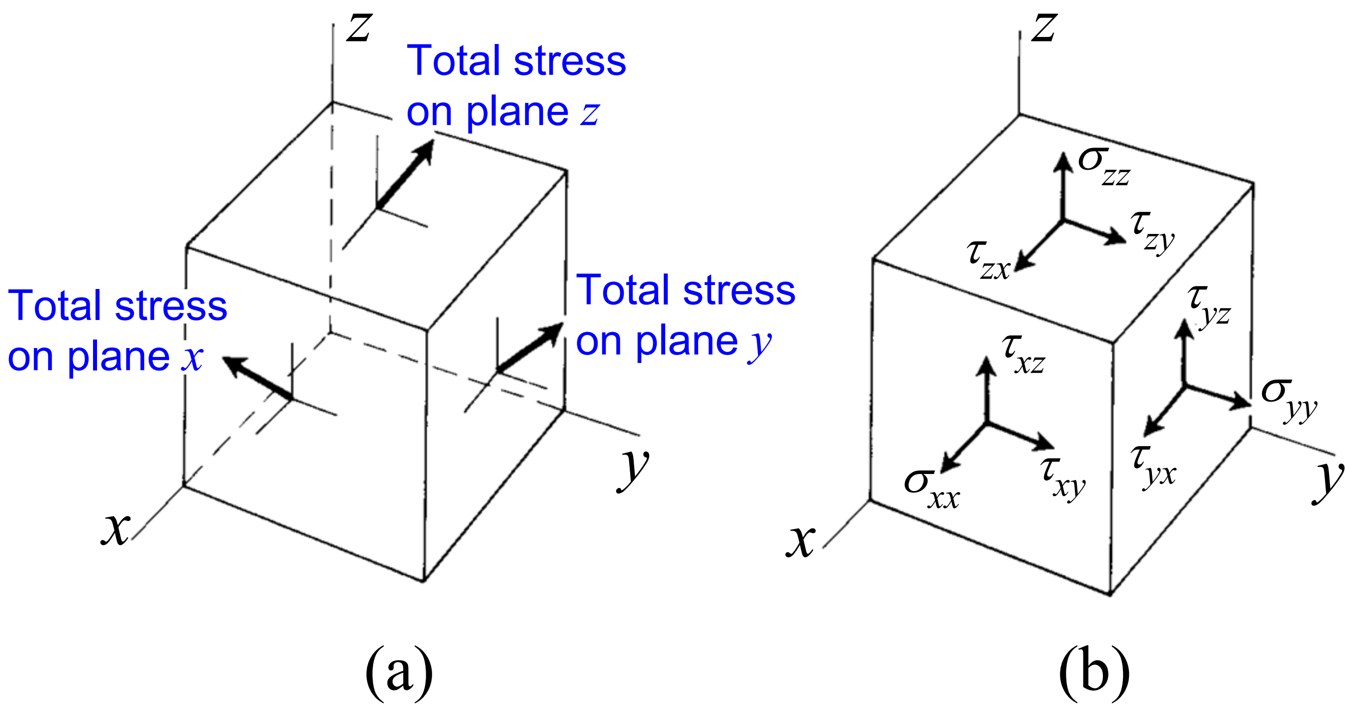
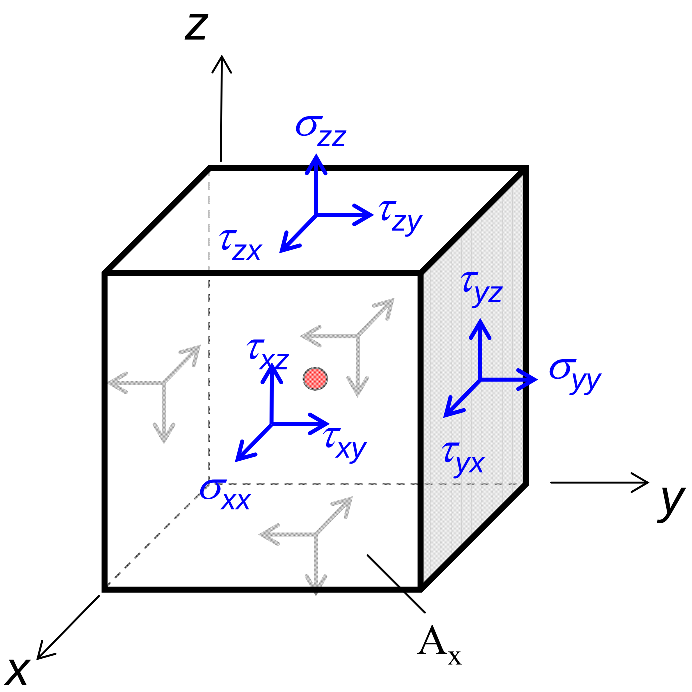


Fig. 1-2. A point-size cube in equilibrium: (a) as extracted showing contact force vectors on forward faces, and (b) stress components. [Figure adapted from W.A. Backofen, Deformation Processing (Addison Wesley, Reading, MA, 1972) p. 3]

Definition of stresses about a point relative to a 3-D coordinate system



6 normal stress components

12 shear stress components

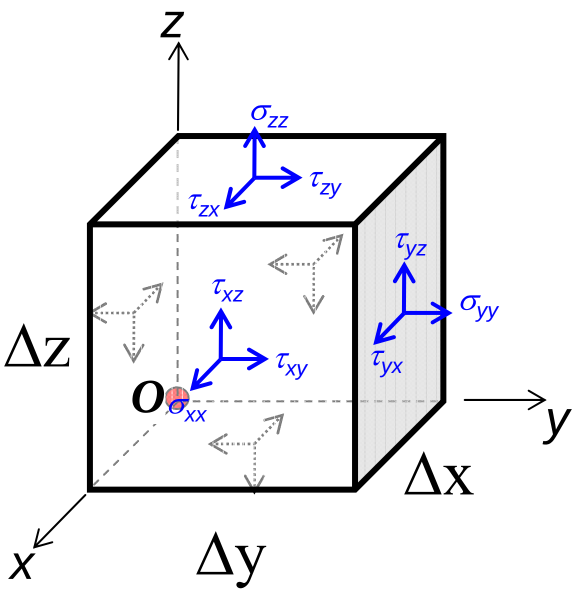
---

18 total components of stress

Points on the back sides of the parallelepiped are not drawn for clarity

The Stress Tensor

- A body is at equilibrium under arbitrary forces.
  - No net forces ($\sum F = 0$)
  - No net torques / moments ($\sum M = 0$)



$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\tau_{ij} = \tau_{ji}$$

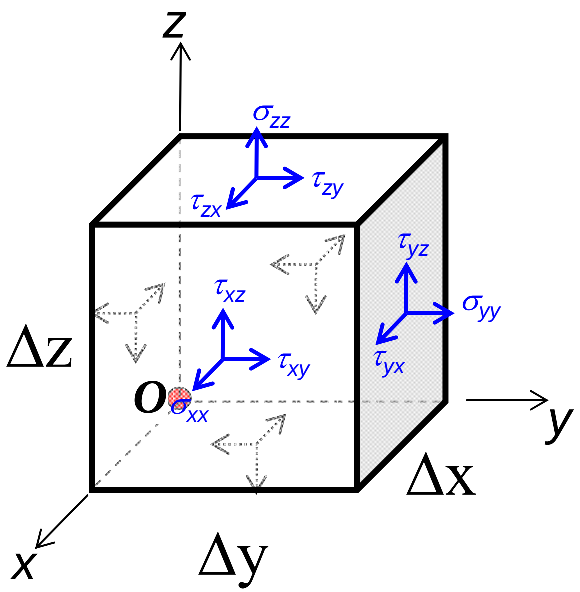
6 independent stress components

The stress tensor is symmetrical

Proof is provided on the next 2 viewgraphs

At equilibrium, ΣF = 0 and ΣM = 0

(there can be no net force or torque)



ΣM = 0

Summation of moments about each axis

z-axis: $\left(\tau_{\mathrm{xy}}\Delta y\Delta z\right)\Delta x=\left(\tau_{\mathrm{yx}}\Delta x\Delta z\right)\Delta y\Rightarrow\tau_{xy}=\tau_{yx}$

y-axis: $\left(\tau_{\mathrm{xz}}\Delta z\Delta y\right)\Delta x=\left(\tau_{\mathrm{zx}}\Delta x\Delta y\right)\Delta z\Rightarrow\tau_{xz}=\tau_{zx}$

x-axis: $\left(\tau_{\mathrm{zy}}\Delta y\Delta x\right)\Delta z=\left(\tau_{\mathrm{yz}}\Delta x\Delta z\right)\Delta y\Rightarrow\tau_{zy}=\tau_{yz}$

Start with

18

Reduce by

-3

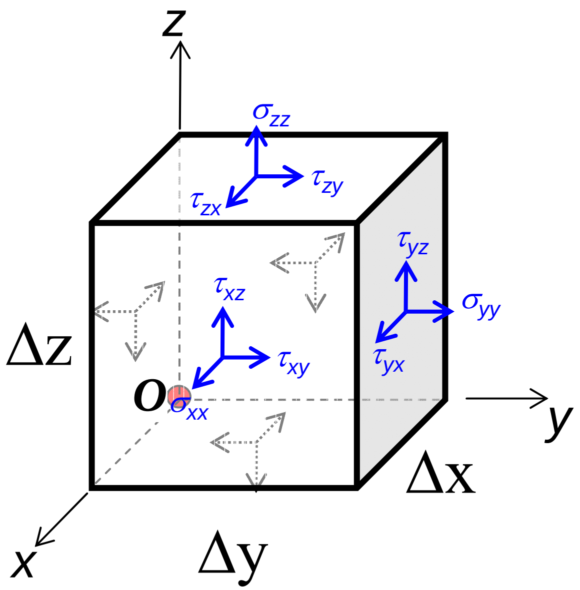
Left with

=15

SUMMATION OF FORCES ON EACH FACE

These relationships allow us to reduce the number of stress components that must be specified to define the state of stress.

$$\Sigma F = 0$$



(on *x*-faces):

$$\sigma_{xx} dydz - \sigma_{-x-x} dydz = 0 \Rightarrow \boxed{\sigma_{xx} = \sigma_{-x-x}}$$

$$\tau_{xy} dydz - \tau_{-x-y} dydz = 0 \Rightarrow \boxed{\tau_{xy} = \tau_{-x-y}}$$

$$\tau_{xz} dydz - \tau_{-x-z} dydz = 0 \Rightarrow \boxed{\tau_{xz} = \tau_{-x-z}}$$

(on *y*-faces):

$$\sigma_{yy} dxdz - \sigma_{-y-y} dxdz = 0 \Rightarrow \boxed{\sigma_{yy} = \sigma_{-y-y}}$$

$$\tau_{yx} dxdz - \tau_{-y-x} dxdz = 0 \Rightarrow \boxed{\tau_{yx} = \tau_{-y-x}}$$

$$\tau_{yz} dxdz - \tau_{-y-z} dxdz = 0 \Rightarrow \boxed{\tau_{yz} = \tau_{-y-z}}$$

(on *z*-faces):

$$\sigma_{zz} dxdy - \sigma_{-z-z} dxdy = 0 \Rightarrow \boxed{\sigma_{zz} = \sigma_{-z-z}}$$

$$\tau_{zx} dxdy - \tau_{-z-x} dxdy = 0 \Rightarrow \boxed{\tau_{zx} = \tau_{-z-x}}$$

$$\tau_{zy} dxdy - \tau_{-z-y} dxdy = 0 \Rightarrow \boxed{\tau_{zy} = \tau_{-z-y}}$$

15

$\boxed{-3}$ =12

12

$\boxed{-3}$ = 9

9

$\boxed{-3}$ = **6**

[18 - 12 = 6 components]

★

The Stress Tensor*

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \text{ or } \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \text{or} \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \cdot & \sigma_{yy} & \tau_{yz} \\ \cdot & \cdot & \sigma_{zz} \end{bmatrix}$$

$$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{yz}, \tau_{xz}$$

A TENSOR is

a group of numbers that represents a physical quantity.

---

i.e., what you get when you relate materials properties to an x,y,z coordinate system.

---

* Read pages 31-36 in Dieter on the stress tensor

5 minute break

Tensor Notation

- The rank* of a tensor determines:
  - Number of components = 3^n where n = rank.
  - Number of direction cosines = n
- The direction cosines are required to transform that physical quantity from one coordinate system to another.

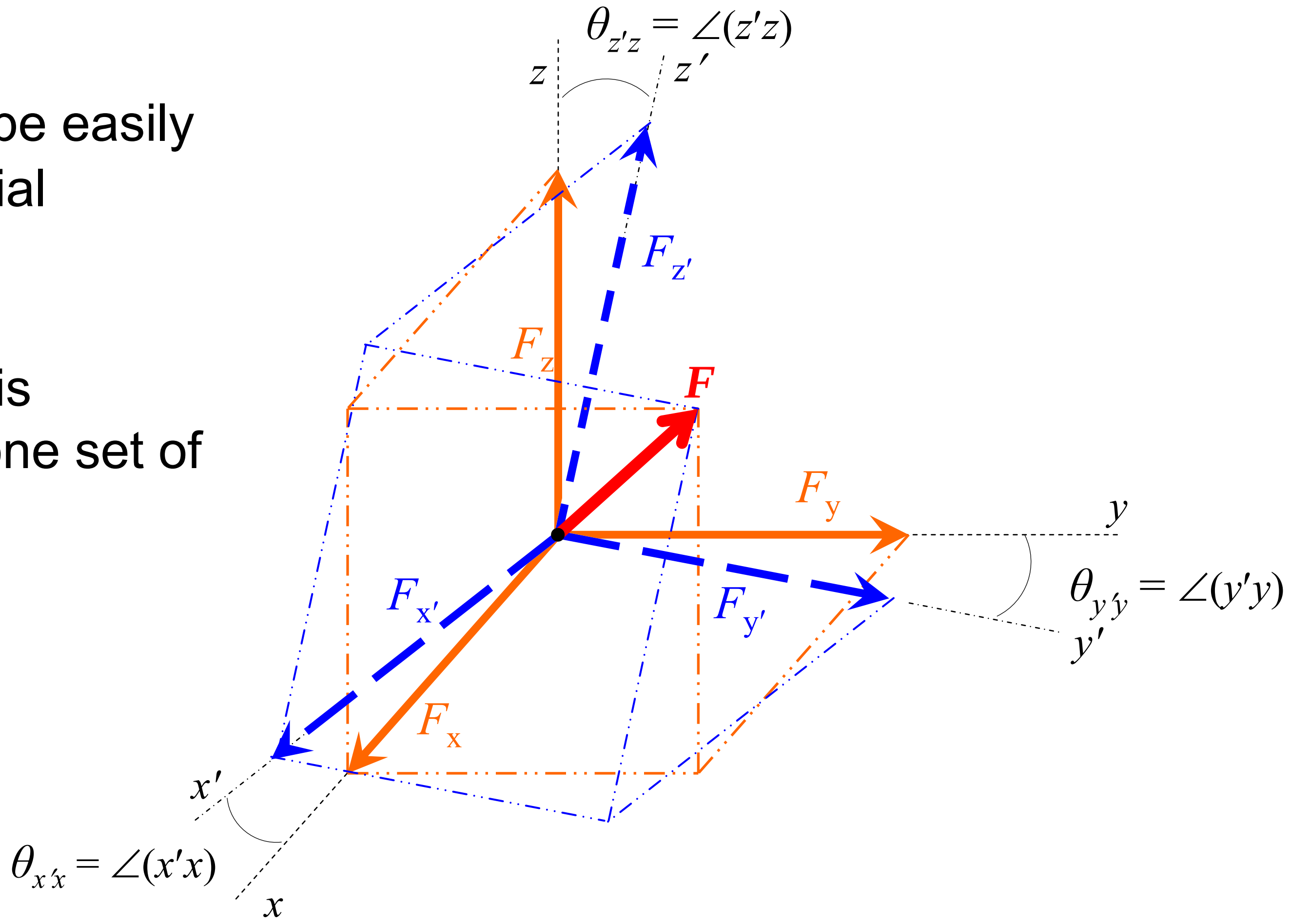
Ranking of Tensors

- Zero rank – *scalars*. Scalars are non-directional.
  - Temperature
  - Density
- First rank tensors – *vectors*. Vectors are directional.
  - Force
  - Area
- Second rank tensors – product of two vectors.
  - Stress (force & area)
  - Strain (displacement & displacement)
- Fourth rank tensors – product of two second rank tensors.
  - Elastic modulus (stress & strain)

★ Transformation of First Rank Tensors

- A vector ***F*** can be easily resolved into axial components.

- Transformation is changing from one set of axes to another.



- Relate material behavior to the symmetry of the materials structure.

- Relate behavior of machine part or specimen to symmetry or shape of part or applied loads.

★

Old Axis

$$\underbrace{\text{Vector } \boldsymbol{F} \text{ resolved onto } xyz}_{F_i}$$

New Axis

$$\left.\begin{array}{c}\text{Vector } \boldsymbol{F}\\ \text{resolved onto}\\ x'y'z'\end{array}\right\} F_{i'}$$

	x	y	z
x'	$\cos(\angle x'x)$	$\cos(\angle x'y)$	$\cos(\angle x'z)$
y'	$\cos(\angle y'x)$	$\cos(\angle y'y)$	$\cos(\angle y'z)$
z'	$\cos(\angle z'x)$	$\cos(\angle z'y)$	$\cos(\angle z'z)$

or

F_i

$F_{i'}$

	x	y	z
x'	$a_{x'x}$	$a_{x'y}$	$a_{x'z}$
y'	$a_{y'x}$	$a_{y'y}$	$a_{y'z}$
z'	$a_{z'x}$	$a_{z'y}$	$a_{z'z}$

All components are related to each other through a series of “direction cosines”

★

$$F_{x'} = a_{x'x}F_x + a_{x'y}F_y + a_{x'z}F_z$$
$$F_{y'} = a_{y'x}F_x + a_{y'y}F_y + a_{y'z}F_z$$
$$F_{z'} = a_{z'x}F_x + a_{z'y}F_y + a_{z'z}F_z$$

$$\begin{bmatrix} F_{x'} \\ F_{y'} \\ F_{z'} \end{bmatrix} = \begin{bmatrix} a_{x'x} & a_{x'y} & a_{x'z} \\ a_{y'x} & a_{y'y} & a_{y'z} \\ a_{z'x} & a_{z'y} & a_{z'z} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

$$F_{i'} = \sum_{j=1}^{3} a_{i'j}F_j = a_{i'j}F_j$$

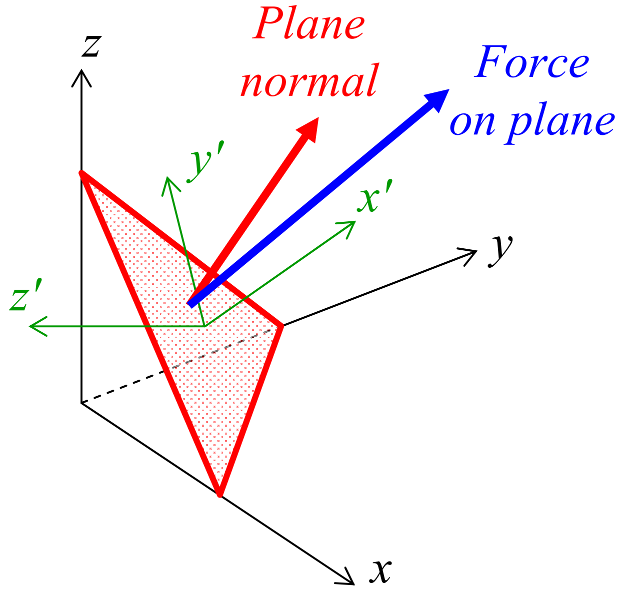
Vector ***F*** transformed to *x′,y′,z′*

Direction cosines relating *x,y,z* to *x′,y′,z′*

Original vector ***F*** resolved onto *x,y,z*

Transformation of Second Rank Tensors

- Second rank tensors denote relationships between two vectors (Force and Area).
- We have to re-orient both vectors.
- Thus, we need two rotation matrices to re-orient stress.



$$\sigma_{i'j'} = \sum_{k=1}^{3}\sum_{l=1}^{3} \sigma_{kl}\, a_{i'k}\, a_{j'l} = \sigma_{kl}\, a_{i'k}\, a_{j'l}$$

Corresponds to area normal to i' direction

Corresponds to force in j' direction

Orientation matrix between re-oriented force and original force

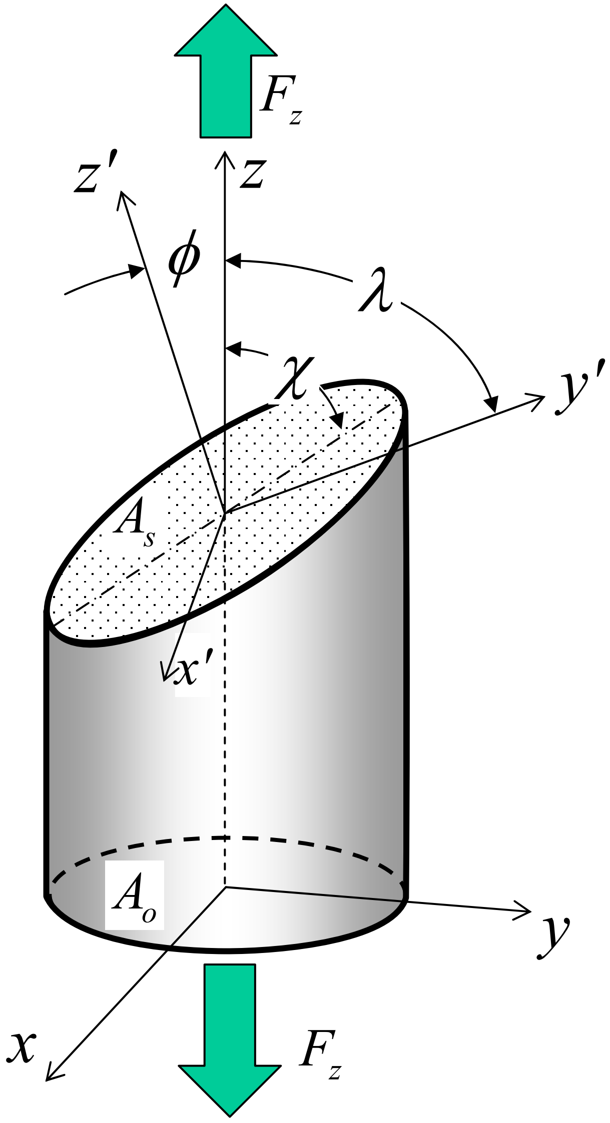
Two direction cosines to transform

Relation to the Deformation of Single Crystals

- Possible to apply a single normal component of stress (σ_{zz}).
- As you will learn later, plastic deformation depends on the shear stress on a specific slip system.

$$\tau_{z'y'} = \frac{F_{y'}}{A_{z'}} = \frac{F_z \cos\lambda}{A_o / \cos\phi} = \sigma_{zz} \cos\lambda \cos\phi$$

- A slip system is a specific crystal plane + specific crystal direction.



$\phi + \chi = 90°$

$\phi + \lambda$ is not necessarily 90°

The transformation equations denoted above can be applied to any first or second rank tensor.

Similar relationships exist for higher rank (order) tensors.

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- S.M. Edelglass: *Engineering Materials Science*, (The Ronald Press Company, New York, 1966).

★

If a second rank tensor is symmetric, it is possible to define a unique set of axes where the tensor will have no off-diagonal components.

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \Rightarrow \sigma'_{ij} = \begin{bmatrix} \sigma_{x'x'} & 0 & 0 \\ 0 & \sigma_{y'y'} & 0 \\ 0 & 0 & \sigma_{z'z'} \end{bmatrix}$$

The new coordinate axes (x', y', z') are called the ***principal axes***

The tensor components ($\sigma_{x'x'}$, $\sigma_{y'y'}$, $\sigma_{z'z'}$) related to them are called the ***principal tensor components***.