HOMEWORK
From Dieter
8-1, 8-6, 8-7, 8-8

Module #6a

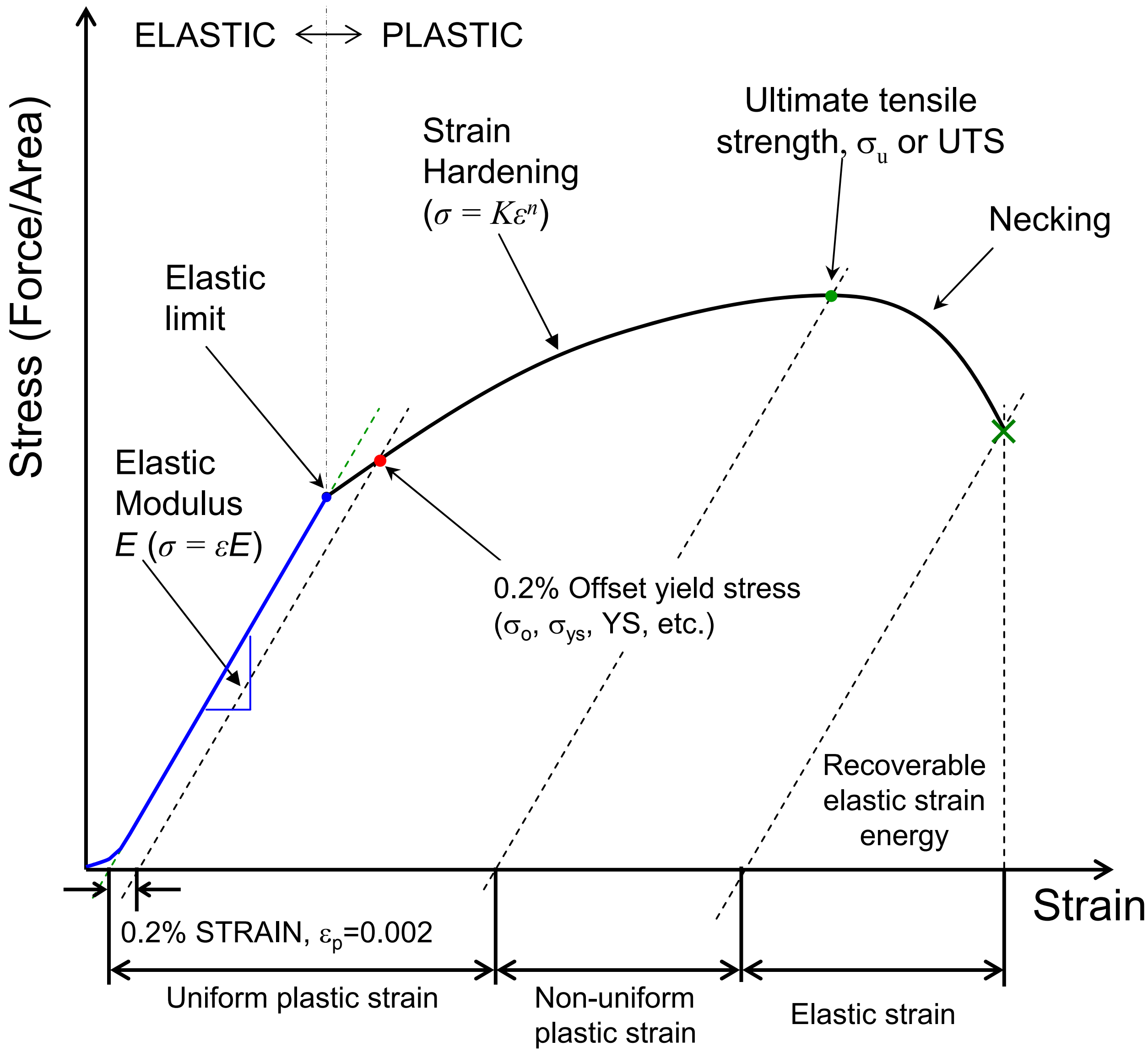
Stress-strain curves
Plastic deformation
Empirical relationships for stress and strain
Criteria for necking

READING LIST

DIETER: Ch. 8, pp. 275-295

Ch. 3 in Meyers & Chawla, 1st ed. (pp. 112-160)
Ch. 1, Pages 1-39 in Courtney

Engineering Stress-Strain Curve in Tension



- Elastic deformation up to elastic limit.
- Plastic deformation after elastic limit.
- Uniform plastic deformation between elastic limit and the UTS.
- Nonuniform plastic deformation after UTS.
- In tension this non-uniform deformation is called necking.

Strain Hardening

- The stress-strain curve (i.e., flow curve) in the region of uniform plastic deformation does not increase proportionally with strain. The material is said to *work harden* (i.e., *strain harden*).

- An empirical mathematical relationship was advanced by *Holloman* in 1945 to describe the shape of the engineering stress-strain curve.

$$\sigma = K\,\varepsilon^n,$$

where is the σ true stress, ε is true strain, K is a strength coefficient (equal to the true stress at $\varepsilon = 1.0$), and n is the strain-hardening exponent. Thus, one can obtain n from a log-log plot of σ versus ε.

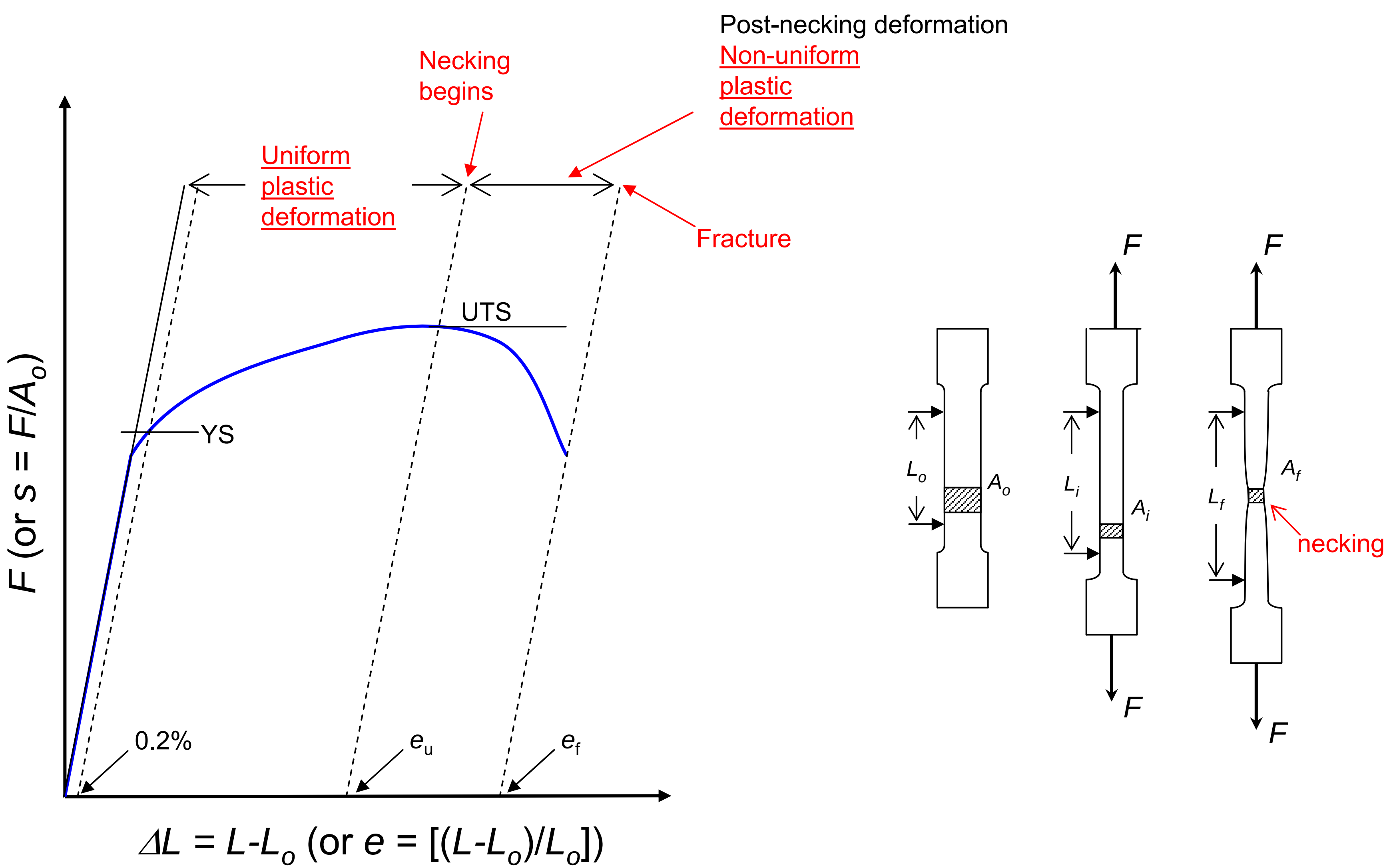
Strain-hardening exponent

$$n = \frac{d(\log \sigma)}{d(\log \varepsilon)} = \frac{d(\ln \sigma)}{d(\ln \varepsilon)} = \frac{\varepsilon}{\sigma}\frac{d\sigma}{d\varepsilon}$$

$n = 0$ for perfectly plastic solids
$n = 1$ for perfectly elastic solids
$n = 0.1 - 0.5$ for most metals

Strain-hardening rate

$$\frac{d\sigma}{d\varepsilon} = n\frac{\sigma}{\varepsilon}$$



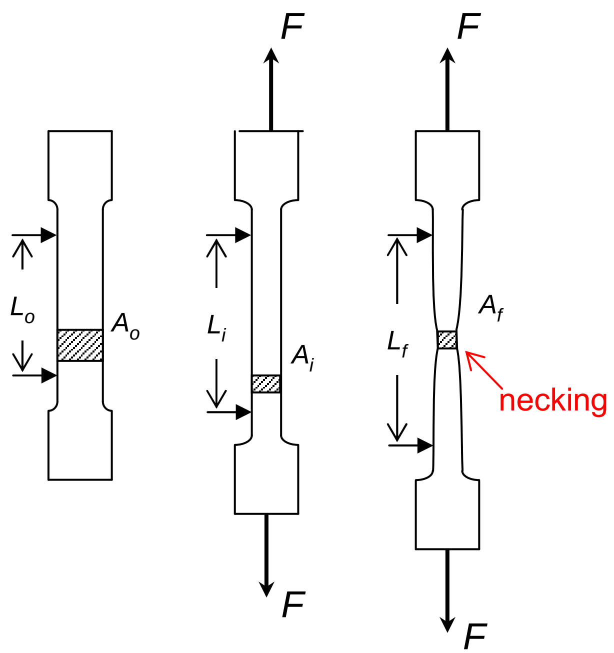
Post-necking deformation
Non-uniform plastic deformation
Necking begins
Uniform plastic deformation
Fracture
UTS
YS
0.2%
e_u
e_f
F (or $s = F/A_o$)
$\Delta L = L-L_o$ (or $e = [(L-L_o)/L_o]$)
F
L_o
A_o
L_i
A_i
L_f
A_f
necking

Why does necking occur?

- We can explain things mathematically by considering strength increases caused by strain hardening and reductions in cross-sectional area caused by the Poisson effect.
- During plastic deformation, the load carrying capacity of the material increases as strain increases due to strain-hardening.
- Strain hardening is opposed by the gradual decrease in the cross-sectional area of the specimen as it gets longer.

Why does necking occur?

- At maximum load (i.e., the UTS on the engineering stress-strain curve) the required increase in stress to deform the material further exceeds its load carrying capacity. This leads to localized plastic deformation or “necking.”



“The material can’t sustain the increase in stress required to continue deforming and work hardening it”

- Necking represents “unstable” flow (deformation)

Criteria for Necking

- Let us start by considering the amount of force *(dF)* that is required to deform a specimen by $d\varepsilon$.

$$F = \sigma A$$

The slope of the stress strain curve is:

$$\frac{dF}{d\varepsilon} = \left[\sigma\left(\frac{dA}{d\varepsilon}\right)\right] + \left[A\left(\frac{d\sigma}{d\varepsilon}\right)\right]$$

NOTE: We are using true stress and strain (i.e., σ, ε) here rather than engineering stress and strain (*s*, *e*)

($d\sigma/d\varepsilon$) is the Work Hardening Rate. It is the slope of the stress-strain curve. It is always positive.

($dA/d\varepsilon$) is the Rate of Geometrical Softening. It is the rate at which the cross-sectional area of the specimen decreases with increasing strain due to constancy of volume. It is always negative.

Criteria for Necking – cont'd

- Local ↓ in A (i.e., deformation) causes that region to strain harden locally (relative to the rest of the cross section). The remainder of the cross section then deforms until a uniform cross-section is re-established.

- The rates balance at the UTS [$(dA/d\varepsilon) = (d\sigma/d\varepsilon)$].

- When $(dA/d\varepsilon) > (d\sigma/d\varepsilon)$, deformation becomes unstable. The material cannot strain harden fast enough to inhibit necking.

Criteria for Necking – cont'd

- The criteria for instability is defined by the condition where the slope of the force distance curve equals zero ($dF = 0$):

$$F = \sigma A$$

$$\text{where}$$

$$F = \text{load,}$$

$$\sigma = \text{true stress,}$$

$$A = \text{area at max load}$$

NOTE: We are using true stress and strain here rather than engineering

$$\boxed{dF = \sigma dA + A d\sigma = 0} \ldots\ldots\ldots\ldots\ldots\ldots(*)$$

Criteria for Necking – cont'd

- Recall that deformation is a constant volume process. Thus:

$$L_o A_o = LA = \text{constant}$$

$$\frac{dL}{L} = -\frac{dA}{A} = d\varepsilon$$

- If we invoke the instability criteria from above (*) then we get:

$$-\frac{dA}{A} = \frac{d\sigma}{\sigma} = d\varepsilon$$

Criteria for Necking – cont'd

- Thus, at the point of tensile instability,

$$\frac{d\sigma}{d\varepsilon} = \sigma$$

When "necking" occurs.

- If we incorporate engineering strain *e*, into the equation presented above, we can develop a more explicit expression:

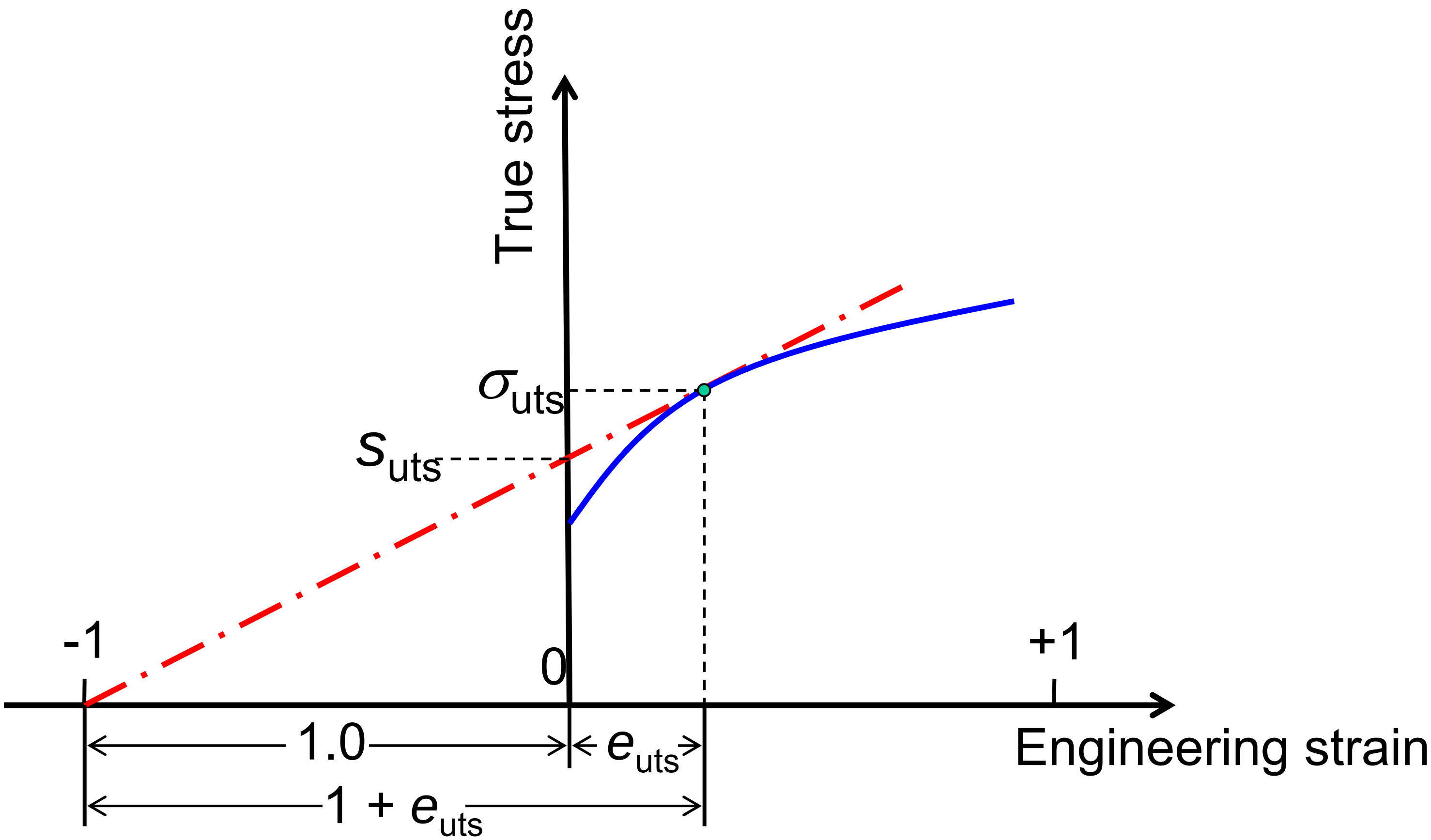
$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{de}\frac{de}{d\varepsilon} = \frac{d\sigma}{de}\frac{dL/L_o}{dL/L} = \frac{d\sigma}{de}\frac{L}{L_o} = \frac{d\sigma}{de}(1+e) = \sigma$$

$$or$$

$$\boxed{\frac{d\sigma}{de} = \frac{\sigma}{(1+e)}}$$

- This is known as Considère's construction.

Considère's Construction



Unstable deformation

- If we substitute the necking criterion,

$$\frac{d\sigma}{d\varepsilon} = \sigma$$

into the equation for the work hardening rate, we get:

$$\frac{d\sigma}{d\varepsilon} = n\frac{\sigma}{\varepsilon} = \sigma$$

which, after re-arranging, becomes:

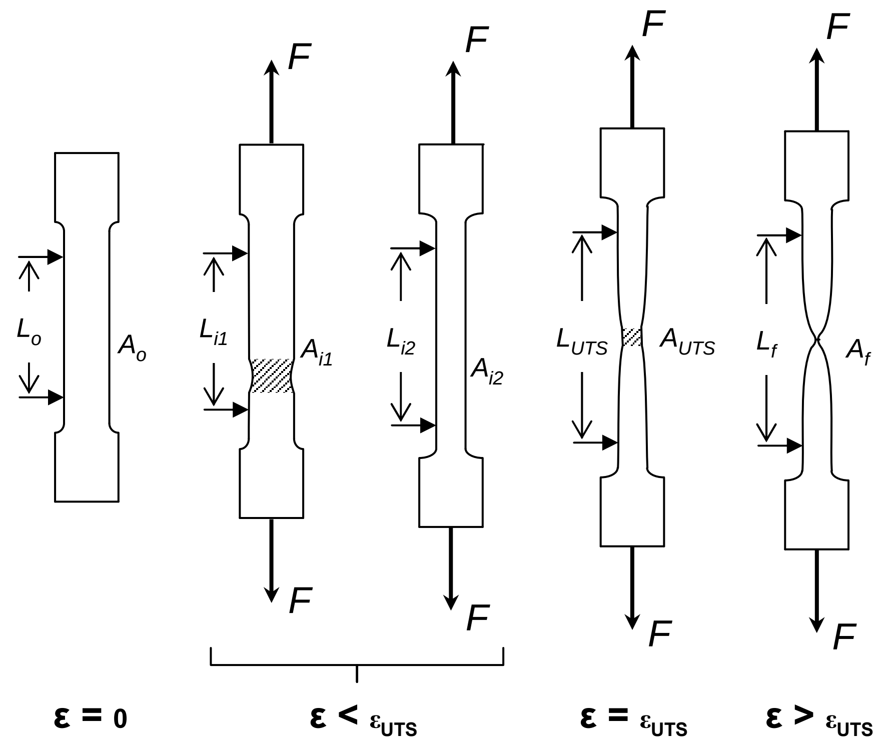
$$n = \varepsilon$$

Work hardening exponent

True uniform strain or Strain at the onset of necking

Process of Necking

(a) During tensile deformation, strain can become localized along the sample length. (b) When strains are less than the UTS, work hardening strengthens the material in the strain localized area relative to the rest of the specimen. (c) The work-hardening rate (WHR) decreases as strain increases. At ε_{UTS} the decrease in cross-sectional area becomes equal to the increase in flow strength due to work hardening. As a result, the localized region (i.e., "neck") becomes permanent. (d) as strain increases, the neck gets bigger until the material fails.



Parameter	**Fundamental Definition**	**Before Necking**	**After Necking**
Engineering stress σ_e	$\sigma_e = s = \frac{F}{A_o}$	$\sigma_e = \frac{F}{A_o}$	$\sigma_e = \frac{F}{A_o}$
True stress σ_t	$\sigma_t = \sigma = \frac{F}{A_i}$	$\sigma_t = \frac{F}{A_i}$	$\sigma_t = \frac{F}{A_{neck}}$
Engineering strain ε_e	$\varepsilon_e = e = \frac{\delta L}{L_o}$	$\varepsilon_e = \frac{\delta L}{L_o}$	$\varepsilon_e = \frac{\delta L}{L_o}$
True strain ε_t	$\varepsilon_t = \ln\frac{A_o}{A_{\min}}$	$\varepsilon_t = \ln\frac{L_i}{L_o} = \ln\frac{A_o}{A_i} = \ln(1+\varepsilon_e)$	$\varepsilon_t = \ln\frac{A_o}{A_{neck}}$

Other Stress-Strain Relationships

- We've already considered the strain hardening exponent. We've noted how it increases with increasing strength and, as you will learn later, decreasing dislocation mobility.

- Stress-strain behavior is also influenced by the rate of deformation (i.e., the strain rate):

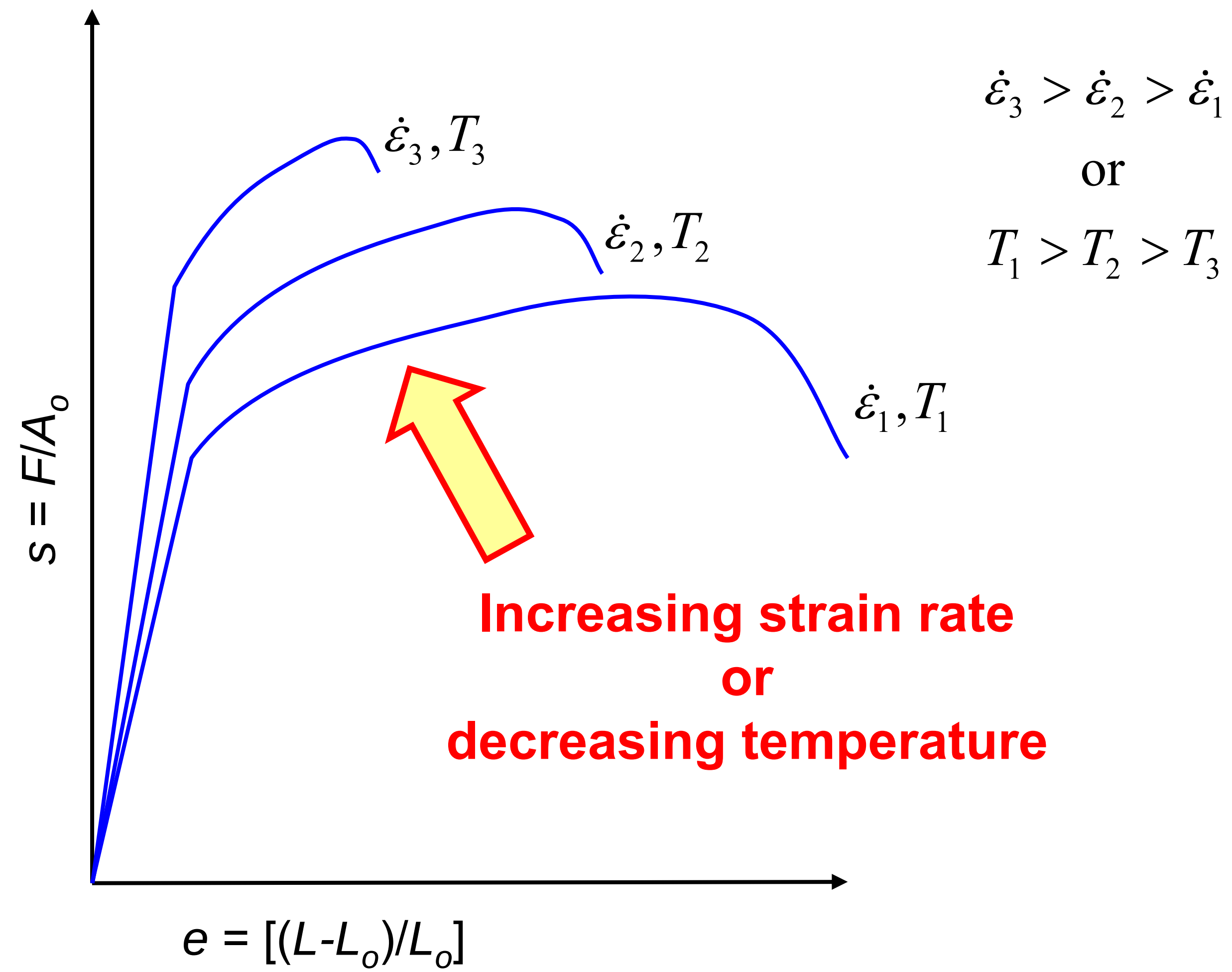
$$\boxed{\sigma = K'\dot{\varepsilon}^m}$$

$\sigma =$ true stress

$K' =$ constant = stress at strain rate of 1 s^{-1}

$\dot{\varepsilon} =$ true strain rate

$m =$ strain-rate sensitivity factor $= d\log\sigma \big/ d\log\dot{\varepsilon}$



Mechanical properties are sensitive to temperature and strain rate.

HOW AND WHY?